Math 671 - Assignment 1 - Due September 6

1. Let Ω be an uncountable set. Show that
$$\mathcal{A} := \{A \subseteq \Omega : A \text{ countable } or A \text{ co-countable}\}$$
is a $\sigma -$ algebra.

2. Let $\mathcal{C} \subseteq \mathcal{P}(\Omega), \sigma(\mathcal{C}) := \bigcap\{\mathcal{A} \subseteq \mathcal{P}(\Omega) : \mathcal{C} \subseteq \mathcal{A}, \mathcal{A} \text{ a } \sigma - \text{algebra}\}$. Prove that $\sigma(\mathcal{C})$ is the smallest $\sigma -$ algebra containing $\mathcal{C}$. (Note that you need to prove the implicit statement that $\sigma(\mathcal{C})$ *is* a $\sigma -$ algebra.)

3. E1.1 from the text.

4. If $\mathcal{C}$ is a π-system, Γ a d-system, and $\mathcal{C} \subseteq \Gamma$, then there is an algebra $\overline{\mathcal{C}}$ such that $\mathcal{C} \subseteq \overline{\mathcal{C}} \subseteq \Gamma$