Math 671 - Assignment 12 (Not due, but look at them)

A couple of these (or something very like them) will be on the exam. Most are just exercises in understanding the notation.

1. E10.3
2. E10.4
3. If X_n and Y_n are submartingales, so is $X_n \vee Y_n = \max(X_n, Y_n)$
4. Let $X_1, X_2, \ldots$ be independent with $\mathbb{E}(X_i) = 0$ and $var(Y_i) = \sigma_i^2$. As usual put $S_n = X_1 + \cdots + X_n$ and write $s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2$. Show that $S_n^2 = s_n^2$ is a martingale.
5. In the version of the Martingale Convergence Theorem we did in class (and the notes), we proved (1) in class. Prove that (1) $\Rightarrow$ (2) $\Rightarrow$ (3). (Hint: consider the submartingale $-X_n$.)