Math 671 - Assignment 2 - Due September 13 (REVISED)

1. Let $(\Omega, \mathcal{A}, \mu)$ be a finitely-additive measure space. Consider the conditions:

   (a) If $F_n \uparrow F$ then $\mu(F_n) \uparrow \mu(F)$

   (b) If $F_n \downarrow F$ then $\mu(F_n) \downarrow \mu(F)$

   (c) If $F_n \downarrow \emptyset$ then $\mu(F_n) \downarrow 0$

   Prove that if μ is finite then these three conditions are equivalent. Which still must hold if μ is infinite?

2. (Don't turn this one in.) If $(\Omega, \mathcal{A}, \mu)$ is a measure space then there is a complete extension $(\Omega, \mathcal{A}^*, \mu^*)$.

3. (This is part of one proof of the Caratheodory extension theorem) Let $(\Omega, \mathcal{A}, \mu)$ be a finitely-additive measure space, and suppose that μ is "continuous from below on $\mathcal{A}$", that is, if $A, A_n \in \mathcal{A}$ and $A_n \uparrow A$ then $\mu(A_n) \uparrow \mu(A)$. Prove that for any nondecreasing sequences A_n, B_n in $\mathcal{A}$ with $\bigcup_n A_n = \bigcup_n B_n$, $\sup_n \mu(A_n) = \sup_n \mu(B_n)$. (In other words, show that the supremum works even if the union is not in $\mathcal{A}$.) (Note: $\mathcal{A}$ need not be a $\sigma -$ algebra in a finitely-additive space!)

4. The "Exercise" on page 25 of the text.