Math 671 - Assignment 3 - Due September 20

1. Let $(\Omega, \mathcal{A})$ be a measurable space, and $f : \Lambda \to \Omega$ any function. Prove that $f^{-1}(\mathcal{A}) = \{f^{-1}A : A \in \mathcal{A}\}$ is a $\sigma -$ algebra on Λ.

2. (Don't turn in) (a) If a topological space $(X, \mathfrak{T})$ is second countable, then the Borel $\sigma -$ algebra $\mathcal{B}_X$ is the $\sigma -$ algebra generated by any basis. What if $(X, \mathfrak{T})$ is *not* second countable? (b) Conclude that if $f_1, f_2, \ldots, f_n$ are real-valued measurable functions then $F = \langle f_1, \ldots, f_n \rangle : (\Omega, \mathcal{A}) \to \mathbb{R}^n$ is measurable into $\mathbb{R}^n$.

3. Let $\mathcal{A} = \sigma(\mathcal{C})$ be a $\sigma -$ algebra on Ω. (a) Show that for any $E \in \mathcal{A}$ there is a countable $\mathcal{C}' \subseteq \mathcal{C}$ (which might depend on E!) such that $E \in \sigma(\mathcal{C}')$. In other words, $\mathcal{A}$ is the union of all $\sigma -$ algebras generated by countable subsets of $\mathcal{C}$. (b) Show that for any $\mathcal{A}$-measurable f there is a countable $\mathcal{C}' \subseteq \mathcal{C}$ such that f is $\sigma(\mathcal{C}')$-measurable.