# Math 671 - Assignment 3a - (also) Due September 20 

1. (Barndorff-Nielsen) Let $\left\{A_{n}\right\}_{n \in \mathbb{N}}$ be a sequence of events, not necessarily independent. Suppose (i) $P\left(A_{n}\right) \rightarrow 0$, and (ii) $\sum_{n} P\left(A_{n} \cap A_{n+1}^{\complement}\right)<\infty$. Prove that $P\left(A_{n}\right.$ i.o. $)=0$.
Why is this a generalization of Borel-Cantelli?
(Hint: for some fixed $\omega \in A_{n}$ i.o., what can we say about the sequence of $A_{n} \mathrm{~s}$ with $\omega \in A_{n} ?$ )
