Math 671 - Assignment 7 - Due Oct. 23

In all the exercises except the last one, X_n is a sequence of random variables and S_n denotes the sum $S_n = X_1 + \cdots + X_n$.

1. If $\sup_n |X_n| \in \mathcal{L}^p$ and $X_n \to X$ a.s. then $X \in \mathcal{L}^p$ and $X_n \to X$ in $\mathcal{L}^p$.

2. For any sequence of random variables X_n and $p \geq 1$,

   (a) If $X_n \to 0$ a.s. then $\frac{S_n}{n} \to 0$ a.s.

   (b) If $X_n \to 0$ in $\mathcal{L}^p$ then $\frac{S_n}{n} \to 0$ in $\mathcal{L}^p$

   (c) Part (b) is false if $0 < p < 1$.

3. If $\frac{S_n}{n} \to 0$ in probability then $\frac{X_n}{n} \to 0$ in probability. Show that this remains true if we replace n by a_n where $\frac{a_{n+1}}{a_n} \to 1$.

4. Prove that the set of real numbers in $[0, 1]$ which do not contain a '2' in the decimal expansion is of Lebesgue measure zero. Deduce from this the existence of two sets A and B each of measure zero such that every real number is representable as a sum $a + b$ with $a \in A$ and $b \in B$