

Math 671 - Assignment 9 - Due Nov. 15

~~(So far the only one I'm sure I want you to hand in is the third problem. I will add one of the others, I haven't decided which one yet.)~~

Turn in the first and third problems.

1. Text E9.2
2. Show that if $\mathcal{C} \subseteq \mathcal{D}$ are sub- σ -algebras of \mathcal{A} and $X \in \mathcal{L}^2$ then $\mathbb{E}((X - \mathbb{E}(X|\mathcal{D}))^2) \leq \mathbb{E}((X - \mathbb{E}(X|\mathcal{C}))^2)$ (That is, the dispersion around the conditional mean shrinks as the σ -algebra grows)
3. Let X, Y random variables and suppose $\mathbb{E}(X|\mathcal{B}) = Y$ and $\mathbb{E}(X^2|\mathcal{B}) = Y^2$. Prove that $X = Y$ a.s.