Math 215 **Final Exam Answers**
May 13, 2008 Name ______________________
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(6) **1.** In the graph given, list the x-values where it is

a. not continuous: -4, 3

b. and where it is not differentiable: -4, -2, 0, 3, 5 (since the picture could be ambiguous, you were given one miss for free)

(10) **2.** Find the limits:

a. $\lim_{x\to 1} \dfrac{x^2-4x+3}{x-1} = -2$

b. $\lim_{x\to 0} \dfrac{\sin(2x)}{x} = 2$

(10) **3.** Find the indefinite integrals:

a. $\displaystyle\int x\sqrt{x^2-2}\,dx$

$\dfrac{1}{3}(x^2-2)^{3/2}+c$

b. $\displaystyle\int (x+1)\sqrt{x}\,dx$

$= \displaystyle\int x^{3/2}+x^{1/2}\,dx = \dfrac{2}{5}x^{5/2}+\dfrac{2}{3}x^{3/2}+c$

This could also be done with integration by parts, but that is much harder.

(15) **4.** Find the derivatives of the functions (do not simplify):

a. $\dfrac{\cos x}{\sqrt{2x+1}}$

$$\frac{-\sin x\sqrt{2x+1}-\cos x\cdot\frac{1}{2}(2x+1)^{-1/2}\cdot 2}{2x+1}$$

b. $x^2+\dfrac{1}{x^2}$

$$2x-2x^{-3}$$

c. $\ln(e^{3x}+\tan(x))$

$$\frac{3e^{3x}+\sec^2 x}{e^{3x}+\tan x}$$

(10) **5.** **a.** Find the tangent line to the curve $y^2 = x^3 + x + 2$ at $(1, -2)$.
Differentiate implicitly: $2yy' = 3x^2 + 1 \implies y' = -1$ at the point (1,-2). The tangent line is $y + 2 = -(x - 1)$.

b. Find a point on the curve where the tangent line is vertical. y' is infinite at $y = 0$. From the original equation, we see this only happens at $x = -1$, so (-1,0) is the point with a vertical tangent line.

(19) **6.** The graph of a differentiable function f on the closed interval $[1, 7]$ is shown above. Let $h(x) = \int_1^x f(t)\,dt$ for $1 \le x \le 7$. [Careful: the graph is of f, the questions concern h.]

a. Find $h(1) = \int_1^1 f(t)\,dt = 0$.

b. Find $h'(4)$. By the Fundamental Theorem, $h'(x) = f(x)$ and $f(4) = 2$ from the graph.

c. On what interval or intervals is the graph of h concave upward? Justify your answer. h is concave up when $h' = f$ is increasing; from the graph this is on the intervals $(1, 3)$ and $6, 7)$.

d. Find the value of x at which h has its minimum on the closed interval $[1, 7]$. Justify your answer.
The points $x = 5, 7$ are where $h' = f$ is zero. But the minimum is at the endpoint $x = 1$: as x increases, the area increases from zero until $x = 5$ where the integral begins to decrease. But the area above the line is greater than the area below the line, so the integral stays positive.

(10) **7.** Johnny dropped his rubber ball; it bounced off the floor with a velocity of 10 feet per second. How high did it go? [The acceleration due to gravity is 32 feet per second per second.]

$$s(t) = -16t^2 + 10t + 0 \quad \text{so} \quad v(t) = -32t + 10$$

The high point is when $v(t) = 0$, so $t = 5/16$. Maximum height is thus $s(5/16) = 25/16$ feet.

(20) **8.** Let R be the region enclosed by the graph of $y = e^x$, the x-axis and the lines $x = -1$ and $x = 2$.

a. Find the area of R.

$$\int_{-1}^{2} e^x\,dx = e^2 - \frac{1}{e}$$

b. Find the volume of the solid generated when R is revolved about the x-axis.

$$\int_{-1}^{2} \pi e^{2x}\,dx = \frac{\pi}{2}\left(e^4 - \frac{1}{e^2}\right)$$