Directions: If your answer doesn’t match the one at the bottom, see if you can figure out why. There’s a great deal of experience to be gained in learning why your mistakes were actually mistakes. If you don’t find out why you were wrong, you’ll keep making the same mistake. On the other hand, maybe you did the problem right, and I just simplified it more than you did.

For each of the following equations, decide whether or not it is an identity. If it is an identity, you must verify it. If it is not an identity, find all values of x for which it holds.

a. \[ 4 \sin^2 x - 2 \left( \sqrt{2} + \sqrt{3} \right) \sin x + \sqrt{6} = 0 \]

b. \[ \sec x - 2/3 = 0 \]

c. \[ \frac{1 + \cos x}{1 - \cos x} - \frac{1 - 2 \cos x}{1 + \cos x} = 4 \cot x \csc x \]

d. \[ \frac{1 + \cos x}{1 - \cos x} - \frac{1 - \cos x}{1 + \cos x} = 4 \cot x \csc x \]

e. \[ \cos \left( \sin^{-1} x \right) = x \]

f. \[ \sin (\pi - x) + \cos (\pi - x) = \sin x - \cos x \]

g. \[ (\tan x + 1) \cot \left( x + \frac{\pi}{4} \right) = 1 - \tan x \]

h. \[ \frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x = \sin \left( x - \frac{\pi}{6} \right) \]

i. \[ 1 - \sin x = \frac{\cos x + 6}{\sin x + 1} \]

j. \[ \frac{\sin 2x}{2 \cos x} - \cos x = 0 \]

k. \[ \tan \left( 2 \cos^{-1} x \right) = \frac{2x \sqrt{1 - x^2}}{2x^2 - 1} \]
Answers

a. 
\[
\begin{align*}
  x &= \pi/4 + 2\pi k \\
  x &= \pi/3 + 2\pi k \\
  x &= 2\pi/3 + 2\pi k \\
  x &= 3\pi/4 + 2\pi k
\end{align*}
\]

\[k = 0, \pm 1, \pm 2, \cdots\]

b. No solutions.

c. \[x = \pi/2 + \pi k, k = 0, \pm 1, \pm 2, \cdots\]

d. Identity.

e. \[x = 1/\sqrt{2}\]

f. Identity.

g. Identity.

h. Identity.

i. No solutions.

j. \[x = \pi/4 + \pi k, k = 0, \pm 1, \pm 2, \cdots\]

k. Identity.