Math 241 Exam 2.

Circle your section (1 pt): 3 4

This is a closed book, closed notes exam, with no calculators or any devices permitted.

Do not write in box below.

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<th>Question</th>
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Put a box around your final answer for each problem.
1. (12 pts) Find the derivatives of the following. You need not simplify.

(a) \( g(x) = \frac{x^2}{10} + \frac{10}{x^2} + \sqrt{10} \)
\[ g(x) = \frac{1}{10} x^2 + 10 x^{-2} + \sqrt{10} \]
\[ g'(x) = \frac{1}{5} x - 20 x^{-3} \]

(b) \( y = \sqrt[4]{2x + 1} + \sqrt[4]{3x + 2} \)
\[ y = \left( (2x+1)^{\frac{1}{4}} + (3x+2)^{\frac{1}{4}} \right)^{\frac{1}{2}} \]
\[ y' = \frac{1}{2} \left( (2x+1)^{\frac{1}{4}} + (3x+2)^{\frac{1}{4}} \right)^{-\frac{1}{2}} \left( \frac{1}{4} (2x+1)^{-\frac{3}{4}} \cdot 2 + \frac{1}{4} (3x+2)^{-\frac{3}{4}} \cdot 3 \right) \]

(c) \( f(x) = x^3 \tan(x) \)
\[ f'(x) = 3 x^2 \tan x + x^3 \sec^2(x) \]

(d) \( u = \frac{\sin(x)}{5x^2 + x} \)
\[ \frac{du}{dx} = \frac{\cos(x) (5x^2 + x) - \sin(x) (10x + 1)}{(5x^2 + x)^2} \]
(e) \( h(x) = (\sec(5x))^3 \)

\[
h'(x) = 3(\sec(5x))^2 \sec(5x) \tan(5x) - 5
= 15(\sec(5x))^3 \tan(5x)
\]

2. (5 pts) Consider the curve \( x^4 + x^2y^3 + y = 13 \). Find the tangent line at the point \((1, 2)\).

Implicit diff:
\[
4x^3 + 2xy^3 + x^2 \cdot 3y^2 \frac{dy}{dx} + \frac{dy}{dx} = 0
\]

\[
(3x^2y^2 + 1)\frac{dy}{dx} = -4x^3 - 2xy^3
\]

\[
\frac{dy}{dx} = \frac{-4x^3 - 2xy^3}{3x^2y^2 + 1}
\]

At \(x=1, y=2\),

\[
\frac{dy}{dx} = \frac{-4-16}{12+1} = \frac{-20}{13}
\]

Slope of normal line is \(\frac{13}{20}\)

Normal line is
\[
y - 2 = \frac{13}{20}(x - 1).
\]
3 (8 pts) A dock is 3 ft higher than the surrounding water. A rope is used to pull in a boat on the water. (Assume the rope is attached to the dock at point that is also 3 ft higher than where it is attached to the boat.) The rope is being pulled (shortened) at 0.4 ft/sec.

**UNITs: ft, s, rad**

Given \( \frac{dy}{dt} = -0.4 \)

(a) How fast is the boat moving when the boat is 4 ft from the dock?

Find \( \frac{dx}{dt} \) when \( x = 4 \).

\[
2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}
\]

\[
2(4) \frac{dx}{dt} = 2(5)(-0.4)
\]

\[
\frac{dx}{dt} = -\frac{4}{8} = -\frac{1}{2} \text{ ft/s}
\]

Boat is moving at \( \frac{1}{2} \text{ ft/s} \)

(b) How fast is angle of the rope changing when the boat is 4 ft from the dock? You may use the angle formed by the rope and the water.

Find \( \frac{d\theta}{dt} \)

\[
\sin \theta = \frac{3}{y}
\]

\[
\cos \theta \frac{d\theta}{dt} = -\frac{3}{y^2} \frac{dy}{dt}
\]

\[
(\frac{4}{5}) \frac{d\theta}{dt} = -\frac{3}{25} (-0.4)
\]

\[
\frac{d\theta}{dt} = -\frac{3}{25} (-0.4) (5)
\]

\[
= \frac{-6}{100} \text{ rad/s}
\]

Angle is changing at \( \frac{6}{100} \text{ rad/s} \)
4. (a) (3 pts) Find the linearization of the function \( f(x) = \frac{1}{x} \) centered at the base point \( x_0 = 2 \).

\[
f'(x) = -\frac{1}{x^2}, \quad f(2) = \frac{1}{2}, \quad f'(2) = -\frac{1}{4}
\]

\[
L(x) = \frac{1}{2} - \frac{1}{4}(x-2).
\]

(b) (3 pts) Suppose an object is moving according to the position function \( s(t) = t^3 - 9t^2 + 15t \). Find the time(s) where this object has zero velocity.

\[
s'(t) = 3t^2 - 18t + 15 \quad \text{set} \quad s'(t) = 0
\]

\[
3(t^2 - 6t + 5) = 0
\]

\[
3(t-1)(t-5) = 0
\]

\[
t = 1 \text{ or } t = 5
\]