Math 241 Quiz 2

Name: Solutions

Section (circle one): 3 4

1. Let \( f(x) = x^2 + 1 \).
   (a) Find the average rate of change of \( f \) over the following three intervals:

   \[
   [1, 2] \quad \frac{f(2) - f(1)}{2 - 1} = \frac{5 - 2}{1} = 3
   \]

   \[
   [1, 1.1] \quad \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.11 + 1) - 2}{1} = 2.1
   \]

   \[
   [1, 1+h] \quad \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{(1+h)^2 + 1 - 2}{h} = \frac{1+2h+h^2 + 1 - 2}{h} = \frac{2h+h^2}{h} = \frac{h(2+h)}{h} = 2 + h
   \]

   (b) Using one of the answers from (a), find the slope of the tangent line (same as instantaneous rate of change) to the graph of \( y = x^2 + 1 \) at the point \( (1, 2) \) (that is, at \( x_0 = 1 \)).

   As \( h \) approaches 0,
   
   \( 2+h \) approaches 2

   (c) Using the answer from (b), find an equation of the tangent line to the graph of \( y = x^2 + 1 \) at the point \( (1, 2) \) (that is, at \( x_0 = 1 \)).

   \( \text{Point and slope} = 2 \).

   \( \text{The tangent line is} \)

   \[ y - 2 = 2(x - 1). \]