1. We want to make a rectangular box such that the length is 3 times the width, of volume 5 cubic meters. The material costs $1/m^2$ except the top of the box costs $2/m^2$. What dimensions of the box would minimize the cost? Be sure to prove that you have an absolute minimum.

\[3x \times y = 5\]

\[3x^2 \times y = 5\]

\[y = \frac{5}{3x^2}\]

Cost is \textit{Top} + \textit{Bottom} + \textit{Front} + \textit{Back} + \textit{Left} + \textit{Right}

\[C = 2(3x^2) + 1(3x^2) + 1(3xy) + 1(3xy) + 1(xy) + 1(xy)\]

\[= 9x^2 + 8xy\]

\[= 9x^2 + 8x\left(\frac{5}{3x^2}\right) = 9x^2 + \frac{40}{3x}\]

\textit{Domain} is \(0 < x\)

\[\frac{dC}{dx} = 18x - \frac{40}{3x^2} \overset{40}{=} 0\]

\[18x = \frac{40}{3x^2}\]

\[x^3 = \frac{40}{3 \times 18} = \frac{20}{27}\]

\[x = \sqrt[3]{\frac{20}{27}}\]

Minimal cost at

width \(x = \frac{5}{\sqrt[3]{27}} \text{ m}\)

height \(y = 3\left(\frac{3^{20}}{27}\right) \text{ m}\)

length \(3x = 3\sqrt[3]{\frac{20}{27}} \text{ m}\)
2. A farmer has 240 ft of fence to make 4 side by side identical rectangular pig pens. What dimensions would maximize the total area?

Constraint: Length of fence
\[ 240 = 2x + 5y \]
\[ 240 - 2x = 5y \]
\[ \frac{240 - 2x}{5} = y \]

Total area is
\[ A = xy = x \left( \frac{240 - 2x}{5} \right) = \frac{1}{5} (240x - 2x^2) \]

Domain is
\[ 0 \leq x \leq 120 \]
where \( 240 - 2x = 0 \)

\[ \frac{dA}{dx} = \frac{1}{5} (240 - 4x) \]
\[ 240 = 4x \]
\[ 60 = x \]

\[ \frac{dA}{dx} \]
\[ 0 \] \[ 60 \] \[ 120 \]

Abs Max

Max area is when \( x = 60, \)
\[ y = \frac{240 - 2(60)}{5} = 24 \]

3. Using Newton's method to find a numerical solution to \( x^4 + x - 1 = 0, \) write down the recursion, and then use it to find \( x_2 \) starting with \( x_0 = 0. \) (Just for fun, if time permits, find a solution to at least 4 decimal places. This requires a calculator or a smart phone.)

Call
\[ f(x) = x^4 + x - 1 \]
\[ f'(x) = 4x^3 + 1 \]
\[ \Rightarrow \]
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]
\[ = x_n - \frac{x_n^4 + x_n - 1}{4x_n^3 + 1} \]

\[ x_0 = 0 \]
\[ x_1 = 0 - \frac{-1}{1} = 1 \]
\[ x_2 = 1 - \frac{1}{\frac{4}{5}} = \frac{4}{5} \]

Just for fun:
\[ x_3 = 0.7312335 \]
\[ x_4 = 0.7245484 \]
\[ x_5 = 0.7244919 \]
\[ 3.7245 \]
3 to 4 decimal places