1. Find the critical points of \( f(x) = x^{2/5}(x - 7) \).

\[
f^'(x) = \frac{7}{5} x^{2/5} - \frac{14}{5} x^{3/5}
\]

\[
f^'(x) = 7x^{2/5} - 14 x^{3/5}
\]

Critical points when

\[
f^'(x) = 0
\]

\[
7x - 14 = 0
\]

\[
x = 2
\]

2. Using the technique from the Extreme Value Theorem for a continuous function on a closed finite interval (compare function values at critical points and endpoints), find the absolute extrema for the function \( f(x) = x^2 - 4x \) on domain \( 0 \leq x \leq 3 \).

\[
f^'(x) = 2x - 4 = 0
\]

\[
x = 2.
\]

Compare critical point(s): \( x = 2, \quad f(2) = 4 - 8 = -4 \) \( \text{MIN} \)

Endpoints \( x = 0, \quad f(0) = 0 \) \( \text{MAX} \)

\( x = 3, \quad f(3) = 9 - 12 = -3 \)

MAX at \( x = 0 \) with value \( f(0) = 0 \)

MIN at \( x = 2 \) with value \( f(2) = -4 \).
3. Prove that the equation \( x^7 + 2x - 5 = 0 \) cannot have more than one solution. (Hint: Let \( f(x) = x^7 + \ldots + 2x - 5 \), and use Rolle’s Theorem.)

Suppose, “by way of contradiction” that \( f(x) = 0 \) did have two or more solutions. Then by Rolle’s Theorem, \( f'(c) = 0 \) for some \( c \) between two of these solutions.

But \( f'(x) = 7x^6 + 2 \) is never 0 because \( 7x^6 \geq 0 \) for all \( x \).

Thus \( f(x) = 0 \) cannot have more than one solution.

4. Find the intervals of increasing/decreasing and local extrema of the function \( f(x) = x^3 - 6x^2 \).

(Find the critical points and then make a 1st derivative diagram.)

\[
\begin{align*}
f'(x) &= 3x^2 - 12x = 0 \\
&= 3x(x - 4) = 0
\end{align*}
\]

Critical points at \( x = 0 \), \( x = 4 \)

\[
\begin{array}{ccccccc}
f'(x): & + & + & 0 & -- & -- & 0 & + & + \\
x: & & & 0 & & & 4 & & \\
\end{array}
\]

Conclusions about \( f \):

Local Max at \( x = 0 \)
Local Min at \( x = 4 \)
Increasing on \((-\infty, 0]\) and \([4, \infty)\)
Decreasing on \([0, 4]\)