If necessary, choose a tail to get the best possible behavior (such as all positive terms, all negative terms, alternating terms, all decreasing terms, or so that certain inequalities are true)

**Easy series?**
- Geometric series?
- p-series?
- Simple telescopic?

- **Yes** → Done
- **No** →

**nth term test**

\[ \lim_{n \to \infty} a_n = 0 ? \]

- **Yes** → Simplify target series via regular comparison or limit comparison?
- **No** → Done, divergent

**Positive/Negative/Alternating/Other?**

- **Positive** → Simplify target series via regular comparison or limit comparison?
- **Negative** → Multiply by $-1$
- **Alternating** → Try absolute convergence. Caution: If \( \sum |a_n| \) diverges, there is no conclusion about \( \sum a_n \).
- **Other**

**Does Alternating series test apply?**

- **Yes** → Done
- **No**
Some useful common limits

An Alternating series test

If

Absolute convergence

If

Inconclusive if

Limit Comparison test for positive series, comparing \( \sum a_n \) with \( \sum b_n \)

If \( a_n \leq b_n \) for a tail, and if \( \sum b_n \) converges then \( \sum a_n \) converges; inconclusive if \( \sum b_n \) diverges.

If \( a_n \geq b_n \) for a tail, and if \( \sum b_n \) diverges then \( \sum a_n \) diverges; inconclusive if \( \sum b_n \) converges.

[For both types of comparison, you might still need to do more work on \( \sum b_n \) using the other tests.]

Ratio test for positive series

If \( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \rho \), a number or \( \infty \) then: \( \sum a_n \) converges if \( \rho < 1 \), diverges if \( \rho > 1 \) or \( \rho = \infty \), and inconclusive if \( \rho = 1 \).

[This test is especially good for series with \( n! \) .]

Root test for positive series

If \( \lim_{n \to \infty} \sqrt[n]{a_n} = \rho \), a number or \( \infty \) then: \( \sum a_n \) converges if \( \rho < 1 \), diverges if \( \rho > 1 \) or \( \rho = \infty \), and inconclusive if \( \rho = 1 \).

[This test is especially good for series with \( (something)^n \).]

Absolute convergence for any series

If \( \sum |a_n| \) converges, then \( \sum a_n \) converges. In fact, we say \( \sum a_n \) converges absolutely.

If \( \sum |a_n| \) diverges, then it is inconclusive about \( \sum a_n \).

[This test is the only hope for series that do not have a tail that is all positive, all negative, or alternating.]

Alternating series test for alternating series

An alternating series is of the form \( \sum (-1)^n u_n \) or \( \sum (-1)^n u_{n+1} \), where \( \{u_n\} \) are positive. If \( \{u_n\} \) are decreasing and \( \lim u_n = 0 \), then \( \sum (-1)^n u_n \) converges. If \( \lim u_n \neq 0 \), then \( \sum (-1)^n u_n \) diverges by nth term test.

Some useful common limits and orders of magnitude

\[
\lim_{n \to \infty} \sqrt[n]{n} = 1, \quad \lim_{n \to \infty} \sqrt[n]{c} = 1, \quad \lim_{n \to \infty} \left( 1 + \frac{x}{n} \right)^n = e^x, \quad \lim_{n \to \infty} \left( \frac{1}{n} \right)^{\frac{1}{n}} = 1, \quad \ln n \ll n^\alpha \ll \beta^n \ll n! \quad (\alpha > 0, \beta > 1)
\]