1. Solve the following differential equations.
   (a) \( \frac{dy}{dx} + 2y = e^x + e^{-x}, \ y(0) = 1 \)

   \[ p(x) = 2 \]
   \[ \int p(x) \, dx = 2x \quad \text{(NDT+C)} \]
   \[ v(x) = e^{2x} \]
   \[ e^{2x} \frac{dy}{dx} + 2e^{2x} y = e^{2x}(e^x + e^{-x}) \]

   \[ \frac{d}{dx} (e^{2x} y) = e^{3x} + e^x \]

   \[ e^{2x} y = \int (e^{3x} + e^x) \, dx \]
   \[ e^{2x} y = \frac{1}{3} e^{3x} + e^x + C \]
   \[ y = \frac{1}{3} e^x + e^{-x} + Ce^{-2x} \]

   Use \( y(0) = 1 \)
   \[ 1 = \frac{1}{3} e^0 + e^0 + Ce^0 \]
   \[ -\frac{1}{3} = C \]

   \[ y = \frac{1}{3} e^{3x} + e^{-x} - \frac{1}{3} e^{-2x} \]
(b) \[ xy' = 3y + x^3 + 3 \]
\[ xy' - 3y = x^3 + 3 \]
\[ y' - \frac{3}{x}y = x^2 + \frac{3}{x} \]
\[ P(x) = -\frac{3}{x} \]
\[ \int P(x) \, dx = \int -\frac{3}{x} \, dx = -3 \ln |x| . \]
\[ V(x) = e^{-3 \ln |x|} = e^{-\ln x^{-3}} = 1x^{-3} . \]
We choose \( V(x) = x^{-3} \)
\[ x^{-3}y' - 3x^{-4}y = x^{-3}(x^2 + \frac{3}{x}) \]
\[ (x^{-3}y)' = x^{-1} + 3x^{-4} \]
\[ x^{-3}y = \int (x^{-1} + 3x^{-4}) \, dx \]
\[ x^3y = \ln |x| + \frac{3x^{-3}}{-3} + C \]
\[ y = x^3\ln |x| - 1 + Cx^3 \]