Math 140   Practice Exam 5

The final covers everything — Lectures 1-30. No Gateway problems. This practice exam covers the material since the last exam. About a third of the exam will be on this material; the other exam problems will be on previous material (see previous practice exams). No decimal answers; all answers must be exact. No calculators.

If you must wear a baseball cap, wear it backwards.

0(14). Find the exact value (no decimals) of
(a) $\cos^{-1}(-\sqrt{3}/2)$  (b) $\tan^{-1}(-\sqrt{3})$
(c) $\sin^{-1}(\sin(\pi/7))$  (d) $\tan(\arcsin(\frac{1}{2}))$

1(6). Find the area (exact value) of a triangle with sides $a = 2$, $b = 3$, and an included angle $\angle C = 25^\circ$.

2(10). An antenna sits on top of a 1000 ft building. From a point level with the base of the building, the building has elevation $20^\circ$ and the antenna has elevation $25^\circ$. How high is the top of the antenna above the ground?

Give exact answers, not decimal answers.

3(8). $a = 3$, $c = 2$, $\angle C = 60^\circ$, solve for all $b$.

4(9). $a = 3$, $\angle C = 60^\circ$, $\angle A = 50^\circ$, solve for $b$.

5(12). $a = 3$, $b = 2$, $\angle B = 30^\circ$, solve for $\angle C$.

6(13). Find the perimeter of a pentagon inscribed in a unit circle (exact answer using radians rather than degrees).

7(3). Convert the given polar coordinates to rectangular coordinates:
(a) $(2, \pi/4)$
(b) $(-1, 2\pi)$
(c) Convert the given rectangular coordinates to polar coordinates: $(\pi, \pi)$

8(6). (a) Convert the polar equation to a rectangular equation: $r^2 = \cos 2\theta$
(b) Convert the rectangular equation to a polar equation: $x^2 - y^2 = 1$

9(14). Find all axes and focus or focal points and draw the graph of $4y^2 - x^2 + 2x - 2 = 0$

10(14). Find all axes and focus or focal points and draw the graph of $4y^2 + x^2 - 2x = 0$

Answers

0. (a) $5\pi/6$  (b) $-\pi/3$  (c) $2\pi/7$  (d) $\pi/4$

1. $3\sin 25^\circ$.

2. $1000 \tan 25^\circ / \tan 20^\circ$ ft.

3. No solutions.

4. $b = \frac{3\sin 70^\circ}{\sin 50^\circ}$.

5. $150^\circ - \sin^{-1}(\frac{3}{4})$ or $\sin^{-1}(\frac{3}{4}) - 30^\circ$.

6. $5\sqrt{2} - 2 \cos \frac{2\pi}{5}$ or $5(\sin \frac{2\pi}{5})/(\sin \frac{3\pi}{10})$.

7. (a) $\left(\sqrt{2}, \sqrt{2}\right)$, (b) $(-1, 0)$, (c) $\left(\pi/2, \pi/4\right)$.

8. (a) $(x^2 + y^2)^2 = x^2 - y^2$.
(b) $r^2(\cos^2 \theta - \sin^2 \theta) = 1$.

9. $4y^2 - x^2 + 2x = 2$
$4y^2 - (x - 2x) = 2$
$4y^2 - (x^2 - 2x + 1) = 1$
$rac{x^2 - (x-1)^2}{1/2} = 1$ vertical hyperbola
$a = 1/2$, $b = 1$, $c = \sqrt{a^2 + b^2} = \sqrt{1/4 + 1} = \sqrt{5}/2$
Shift: $(0, \sqrt{5}/2)$ right 1 get major axis: $(1, \sqrt{5}/2)$
Shifting $(-1, 0)(1, 0)$ right 1 get minor axis: $(0, 0)(2, 0)$
Shifting $x = -1$ up and right 1 gives directrix: $x = 0$.
Shift $(0, \pm \sqrt{5}/2)$ right 1 get foci: $(1, -\sqrt{5}/2), (1, \sqrt{5}/2)$.
Now graph (see below left). The foci are $\approx (-1.11)$ and $(1.11)$.

10. $4y^2 + (x^2 - 2x + 1) = 1$
$4y^2 + (x - 1)^2 = 1$
$rac{x^2 - (x-1)^2}{1/2} = 1$ horizontal ellipse
c $= \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{2}} \approx 0.87$
Shift: right 1 unit.
Shift $(-1, 0)(1, 0)$ right 1, get major axis: $(0, 0)(2, 0)$.
Shift $(0, \sqrt{3}/2)(0, \sqrt{3}/2)$ right 1, get minor axis: $(1, \sqrt{3}/2)(1, -\sqrt{3}/2)$.
Shift $(\pm \sqrt{3}/2, 0)$ right 1 get foci: $(1 - \sqrt{3}/2, 0), (1 + \sqrt{3}/2, 0)$.
Now graph (below right). The foci are $\approx (-1.3, 0)$ and $(1.87, 0)$.