Name:

UH ID:

**Directions:** Print your name and ID number. Clearly mark your answers. You may write on the exam, but will only be graded on the answer you mark. If you need to change answers, please completely erase or cross out the old answer and write and circle your final choice.

The passing score is 26.
A gutter is constructed by bending a rectangular sheet of metal into right angles at the dotted lines as shown here. The sheet of metal measures 6 inches by 100 inches and the bends are made at measurements of $x$ inches as shown. Give a formula for the volume $V$ of the resulting gutter (in inches$^3$), as a function of $x$.

(A) $V = 100x(6 - 2x)$  
(B) $V = (100 - 2x)(6 - 2x)x$  
(C) $V = 600 - 2x$  
(D) $V = 100x(6 - 4x)$  
(E) $V = 6x(100 - 2x)$

If $|x + 3| < 4$ and $|x - 5| = 6$, what is $x$?

(A) 1  (B) -1  (C) 2  
(D) 0  (E) 8
Here are three statements about the polynomial 
\[ p(x) = 9 - x + x^2 - x^3 : \]

I. The degree of \( p(x) \) is 3.
II. The leading coefficient of \( p(x) \) is \(-1\).
III. The graph of \( p(x) \) has the end behavior pictured here:

Which of the statements are true?

(A) None of these \hspace{1cm} (B) Only I \hspace{1cm} (C) Only II
(D) Only III \hspace{1cm} (E) Only I and II

Shown here is the graph of the polynomial \( p(x) \). Which of the choices below is the only possible formula for \( p(x) \)?

(A) \( p(x) = (x + 2)(x - 3)(x - 5) \)
(B) \( p(x) = -(x + 2)(x - 3)^2(x - 5)^2 \)
(C) \( p(x) = (x + 2)(x - 3)(x - 5)(8x^2 + 1) \)
(D) \( p(x) = -(x + 2)(x - 3)(x - 5)(8x^2 + 1) \)
(E) \( p(x) = (x + 2)(x - 3)^2(x - 5)^2 \)
Which of these functions has a graph that is the same as the graph of \( f(x) \), but shifted 6 units down?

(A) \( f(x) - 6 \)  \hspace{1cm} (B) \( f(x - 6) \)  \hspace{1cm} (C) \( 6f(x) \)

(D) \( f(x) + 6 \)  \hspace{1cm} (E) \( f(x + 6) \)

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Perform the polynomial division:

\[
\begin{array}{c|ccccc}
& x^2 & x & -2 & \\ \hline x^3 & -4x^2 & -7x & +10 & \\
\end{array}
\]

Give both the quotient \( q(x) \) and the remainder \( r(x) \).

(A) \( q(x) = x - 3 \) and \( r(x) = -12x + 4 \)

(B) \( q(x) = x - 3 \) and \( r(x) = -2x + 4 \)

(C) \( q(x) = x - 4 \) and \( r(x) = -3 \)

(D) \( q(x) = x - 4 \) and \( r(x) = 2x + 1 \)

(E) \( q(x) = x - 5 \) and \( r(x) = 0 \)
Listed here are three functions. Which of these is a rational function?

I. \( \frac{\sqrt{2} x^2 + 4}{2x - 5} \)  
   II. \( \frac{3 + x + e^x}{4x + 3x^2} \)  
   III. \( \sqrt{16 - x^2} \)

(A) Only I  (B) Only II  (C) Only III  
(D) Only I and II  (E) None of them

What type of function is \( f(x) = \log_3 x + 5 \)?

(A) exponential  (B) logarithmic  
(C) rational  (D) linear  
(E) cubic
Compute $\log_8 2$.

(A) $\frac{1}{4}$ \hspace{1cm} (B) $\frac{1}{16}$ \hspace{1cm} (C) 2

(D) $\frac{1}{3}$ \hspace{1cm} (E) 4

Shown here is the graph of the function $f(x)$. Which of the choices below shows the graph of $f^{-1}(x)$ in red?

A \hspace{1cm} B \hspace{1cm} C

D \hspace{1cm} E
Shown here is a graph of $y = f(x)$. What is $f^{-1}(-5)$?

(A) $-\frac{1}{4}$  (B) $\frac{1}{4}$

(C) 5  (D) 4

(E) $-5$

Which of the functions below is an exponential function with base $e$?

(A) $e^x$  (B) $xe$  (C) $\ln x$

(D) $\log_e x$  (E) $e^2$
Which of the functions below is a logarithmic function with base 10?

(A) \( \log_x 10 \)  
(B) \( 10^x \)  
(C) \( \log_{10} x \)

(D) \( \ln 10 \)  
(E) \( \ln x \)

Solve for \( x \) in the equation \( 2 = 2 \log_3 (x - 1) + 6 \).

(A) \( \frac{10}{9} \)  
(B) \( -2 \)  
(C) \( \frac{244}{243} \)

(D) \( 5 \)  
(E) \( 4 \)
Shown here are several graphs. Which is the graph of \( \log_3 x \) and which is the graph of \( \left( \frac{3}{2} \right)^x \)?

(A) I is \( \log_2 x \) and IV is \( \left( \frac{3}{2} \right)^x \)

(B) II is \( \log_3 x \) and III is \( \left( \frac{3}{2} \right)^x \)

(C) II is \( \log_3 x \) and IV is \( \left( \frac{3}{7} \right)^x \)

(D) I is \( \log_3 x \) and III is \( \left( \frac{3}{7} \right)^x \)

(E) IV is \( \log_3 x \) and I is \( \left( \frac{3}{2} \right)^x \)

Express \( \ln 11 \) in terms of base 3 logarithms.

(A) \( \log_3 \frac{e}{11} \)  

(B) \( \log_3 11 \)  

(C) \( \frac{\log_3 e}{\log_3 11} \)

(D) \( 11 \log_3 e \)  

(E) \( \frac{\log_3 11}{\log_3 e} \)
Find a formula for the inverse of \( f(x) = (x + 4)^3 - 2 \).

(A) \( \sqrt[3]{x + 2} - 4 \)  
(B) \( (x + 4)^{-3} - 2 \)  
(C) \( (x + 4)^{-1/3} + 2 \)  
(D) \( \sqrt[3]{x} - 2 \)  
(E) \( \sqrt[3]{x} + 2 \)

Here is a list of values for the function \( k(x) \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k(x) )</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Compute \( k^{-1}(2) \).

(A) 0  
(B) 4  
(C) -2  
(D) \( -\frac{1}{2} \)  
(E) It is undefined
What is the range of the function \( W(x) = -(x - 2)^2 + 5 \)?

(A) \([5, \infty)\)  
(B) \([2, \infty)\)  
(C) \((-\infty, 5]\)

(D) \((-\infty, \infty)\)  
(E) \((-\infty, 2]\)

Listed here are three functions. Which of these is a polynomial?

I. \(x^{2/3} + 5x^{5/3} + 7\)  
II. \(\frac{x - 1}{x + 1}\)

III. \(3 + 4x^2 - \frac{2}{3}x^3\)

(A) None of them  
(B) Only I  
(C) Only II  
(D) Only III  
(E) Only I and III
Find all vertical and horizontal asymptotes of the rational function $f(x)$, which has been factored for your convenience.

$$f(x) = \frac{5x^2 + 11x - 36}{x^3 - x^2 - 6x} = \frac{(5x - 9)(x + 4)}{x(x + 2)(x - 3)}$$

(A) $x = 0, x = -2, x = 3, y = 0$

(B) $x = \frac{9}{5}, x = -4$

(C) $x = \frac{9}{5}, x = -4, y = 0$

(D) $x = 0, x = -2, x = 3, y = 5$

(E) $x = 5, y = 0, y = -2, y = 3$

What is the remainder when we divide $x + 1$ into $x^9 - x^2 + 3$? (HINT: Use the Remainder Theorem).

(A) 1  (B) -1  (C) 3

(D) 2  (E) -4
Put these numbers in order from least to greatest:

\[ e^{-2} \quad \ln \frac{1}{e} \quad e^0 \]

(A) \( e^{-2} < \ln \frac{1}{e} < e^0 \)
(B) \( e^{-2} < e^0 < \ln \frac{1}{e} \)
(C) \( \ln \frac{1}{e} < e^{-2} < e^0 \)
(D) \( \ln \frac{1}{e} < e^0 < e^{-2} \)
(E) \( e^0 < e^{-2} < \ln \frac{1}{e} \)

Here are three statements about graphs of polynomial and rational functions:

I. The graph of a degree 3 polynomial may have up to 3 vertical asymptotes.
II. The graph of a rational function may have both a horizontal and an oblique asymptote.
III. The graph of a rational function may cross a vertical asymptote.

Which of the statements are true?

(A) None of these \hspace{1cm} (B) Only I \hspace{1cm} (C) Only II
(D) Only III \hspace{1cm} (E) Only II and III
Solve the inequality \( x^2 + 8x + 16 < 0 \).

(A) \( x < 4 \)  \hspace{1cm} (B) \( x > 4 \)  \hspace{1cm} (C) \( x < -4 \)

(D) \( x > -4 \)  \hspace{1cm} (E) There are no solutions

Simplify the expression \( \log_4 (4^{x-3}) \).

(A) \( 4x - 12 \)  \hspace{1cm} (B) \( x - 3 \)  \hspace{1cm} (C) \( 4^x - 64 \)

(D) \( 64 \)  \hspace{1cm} (E) \( \log_4 (x - 3) \)
Which of the five graphs below is the graph of

\[ y = \frac{x + 1}{(x + 2)^2} \]

(The y-scales have been intentionally omitted.)

[Graphs A, B, C, D, E are shown]

Let \( f(x) \) be a one-to-one function with domain \((-2, 4)\) and range \((3, 12)\). What is the range of \( f^{-1}(x) \)?

(A) \((-\frac{1}{2}, \frac{1}{4})\)  (B) \((\frac{1}{4}, \frac{1}{12})\)  (C) \((-\frac{1}{4}, -\frac{1}{12})\)

(D) \((-2, 4)\)  (E) \((3, 12)\)
Express \( 2 \ln a - \ln (b - c) \) as a single logarithm.

(A) \( \ln (a^2 - b - c) \)  
(B) \( \ln (2a - b - c) \)

(C) \( \ln \left( \frac{2a}{b - c} \right) \)  
(D) \( \ln (ab - ac) \)

(E) \( \ln \left( \frac{a^2}{b - c} \right) \)

Here are three statements about the parabola which is the graph of the equation \( y = \frac{2}{3}(x + 5)^2 - 1 \).

I. The vertex of the parabola is at \((-5, -1)\).
II. The parabola is wider than \( y = x^2 \).
III. The parabola opens upward.

Which of the three statements are true?

(A) Only I  
(B) Only II  
(C) Only III

(D) Only II and III  
(E) All three are true
The graph of the equation \( y = 2x^2 + 8x + 3 \) is a parabola. What are the coordinates of its vertex?

(A) \((-2, -5)\)  \hspace{1cm} (B) \((-4, -13)\)  \hspace{1cm} (C) \((-2, -1)\)

(D) \((-4, -1)\)  \hspace{1cm} (E) \((4, -13)\)

Find all values of \( t \) that solve the equation:

\[
\frac{3t - 4}{t} + \frac{t}{2} = t
\]

(A) \( t = \frac{4}{7} \) \hspace{1cm} (B) \( t = 1 \) and \( t = 5 \)

(C) \( t = 2 \) and \( t = 4 \) \hspace{1cm} (D) \( t = -1 \) and \( t = \frac{2}{3} \)

(E) There are no solutions
Find all values of $x$ that solve the equation:

$$\sqrt{x-1} + 3 = x$$

(A) $x = 2$ and $x = 5$  
(B) $x = 1$

(C) $x = 5$  
(D) $x = 1$ and $x = 10$

(E) There are no solutions

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An office cubicle is being constructed against an existing wall of the building. Two walls of length $x$ and one wall of length $y$ are required, as shown here. If 36 feet of material are available, what dimensions should the cubicle have so that it encloses the largest possible area?

(A) $x = 15$ ft and $y = 6$ ft  
(B) $x = 18$ ft and $y = 14$ ft

(C) $x = 10$ ft and $y = 16$ ft  
(D) $x = 12$ ft and $y = 12$ ft

(E) $x = 9$ ft and $y = 18$ ft
What is the equation of the parabola graphed here?

(A) \( y = (x - 3)^2 - 1 \)
(B) \( y = (x - 1)^2 + 3 \)
(C) \( y = -(x - 1)^2 + 3 \)
(D) \( y = -(x + 1)^2 + 3 \)
(E) \( y = (x - 1)^2 - 3 \)

Which of the five graphs below is the graph of the polynomial
\[
y = -x^3 - 3x^2 + x + 3 = -(x - 1)(x + 1)(x + 3)
\]
(The \( y \)-scales have been intentionally omitted.)

A

B

C

D

E
Solve for $x$ in the equation: $\log_9 x = -2$.

(A) $-3$  
(B) 3  
(C) $\frac{1}{81}$  
(D) $\frac{1}{3}$  
(E) There is no solution

Solve for $x$ in the equation: $(\frac{1}{4})^{3-5x} = 4$.

(A) $x = 1$  
(B) $x = -3$  
(C) $x = \frac{4}{9}$  
(D) $x = \frac{1}{5}$  
(E) $x = 3$
Find all values of \( x \) that satisfy the equation:

\[ \log_7(x + 1) - \log_7 x = \log_7 5 \]

(A) \( x = \frac{1}{4} \)  
(B) \( x = 1 \)  
(C) \( x = \frac{4}{7} \)  
(D) \( x = 49 \)  
(E) There are no solutions

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Expand the logarithmic expression: \( \ln \frac{\sqrt{m}}{p^3 + w^5} \).

(A) \( \frac{1}{2} \ln m - \ln(p^3 + w^5) \)  
(B) \( \frac{1}{2} \ln m - \frac{1}{2} \ln(p^3 + w^5) \)  
(C) \( \frac{1}{2} \ln m - 3 \ln p - 5 \ln w \)  
(D) \( \frac{1}{2} \ln m - 3 \ln p + 5 \ln w \)  
(E) \( \frac{1}{2} \ln m - \frac{3}{2} \ln p - \frac{5}{2} \ln w \)