Modular forms reading list

Several textbooks and other materials are described below. I have organized them in a hopefully helpful way.

Standard textbooks

First, a list of some textbooks that I would call the “main” texts for one first learning about modular forms.

- Diamond–Shurman, *A first course in modular forms*: a recent (read “modern”) textbook that gives a nice introductory account of the subject. I would say much of the beginning of the course will be based on parts of this book.
- Miyake, *Modular forms*: a standard (read “classic”) text on the subject.
- Shimura, *Introduction to the arithmetic theory of automorphic functions*: probably more of a standard, classic text than Miyake’s book. Probably where most practitioners of the subject first learned it.

Where you might first read about modular forms

These are some less extensive places to read about modular forms.

- Serre, *A course in arithmetic*, Chapter 7: probably where most people first read about modular forms. It is a brief, but characteristically nice, discussion of modular forms on \( SL(2, \mathbb{Z}) \).
- Koblitz, *Introduction to elliptic curves and modular forms*: Chapter III gives an introduction to modular forms and Chapter IV is even about half-integer weight modular forms.
- Silverman, *Advanced topics in the arithmetic of elliptic curves*, Chapter I: 90 pages telling you about the basics of what you need to know.

Surveys

- Diamond–Im, *Modular forms and modular curves*: a very nice overview of much of what we will cover in this course. This is a great article to have on hand as it covers briefly the topics covered more in depth in the following sections.
- Bump et al., *An introduction to the Langlands program*: a survey of material around modular forms with a view towards introducing the Langlands program.

Specific topics

There are several ways of thinking about modular forms and because these are usually how one comes across them in the literature, I want to make sure to introduce them to you. Unfortunately, these do not occur in the usual introductory textbooks, so here are separate sources for these specific topics.

Via algebraic geometry

Modular forms are sections of certain universal line bundles defined on moduli spaces over \( \mathbb{Z} \). These sources contain relevant material.

- Katz, *p-adic properties of modular schemes and modular forms*: this is where the theory began. Katz is a very good writer, but that doesn’t prevent the material from being difficult!
- Hida, *Geometric modular forms and elliptic curves*: a recent account of the subject. Bonus: begins with a crash course in algebraic geometry! (Note: the second edition contains expanded content).

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1Prepared for Math 847 at the University of Wisconsin–Madison by Robert Harron.
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• Katz–Mazur, *Arithmetic moduli of elliptic curves*: doesn’t really talk about modular forms, but is a standard reference on the underlying moduli problems. If you’re hardcore (i.e. the term “Artin stack” gets you up in the morning), you can also check out Brian Conrad’s article with nearly the same title.

**Automorphic forms**

Modular forms are a special case of automorphic forms. These more general objects are very important tools these days in number theory.

• Gelbart, *Automorphic forms on adele groups*: an excellent book that begins by reviewing relevant facts about classical modular forms and then explains the transition to automorphic forms and representations on $GL(2, \mathbb{A})$.

• Bump, *Automorphic forms and representations*: what I would call the standard first text on the subject these days.

• Borel–Casselman, eds, *Corvallis proceedings, i.e. Automorphic forms, representations, and $L$-functions*: the standard reference for the subject is these (massive) proceedings from the late 1970s. These are available for free on the AMS website.

• Borel, *Introduction to automorphic forms*: this is a nice little article that gives you an idea as to why things are defined the way they are. It is in another proceedings available for free on the AMS website (*Algebraic groups and discontinuous subgroups*). These proceedings contain some other nice articles.

• Gel’fand–Graev–Pyatetskii-Shapiro, *Representation theory and automorphic functions*: an old-school textbook on the subject. One of the originals.

**Via group (co)homology**

Modular forms can actually be viewed as elements in the group cohomology of discrete subgroups of $SL(2, \mathbb{R})$. This is what allows us to compute with them in Sage, etc.

• Stein, *Modular forms: a computational approach*: this book is in fact an alternative for a basic textbook on modular forms. It’s in this section of this list because it contains material on modular symbols and group cohomology.

• Shimura, *Introduction to the arithmetic theory of automorphic functions*, Chapter 8: this is a chapter in the aforementioned book by one of the mathematicians whose name is central to this view of modular forms.

• Hida, *Elementary theory of $L$-functions and Eisenstein series*, Chapter 6: a (hopefully) easier to read account than Shimura’s. This book, as with all of Hida’s books, covers a wealth of interesting material not available anywhere else in textbook form.

**Material with which I am less familiar**

This section is material I have not personally looked at much, but which I suspect is still quite good. The list is not exhaustive, so if there are any glaring omissions, I’ll happily add them.

• Knapp, *Elliptic curves*, Chapters VIII, IX, XI: I’ve found many of Knapp’s books and articles to be above par. Why haven’t I looked at this one??

• Milne, *Modular functions and modular forms*: obviously on his website, with many of his other great notes.

• Lang, *Introduction to modular forms*: an interesting book that covers important material not usually found in a book with this title.
• Iwaniec, *Topics in classical automorphic forms* and *Spectral methods of automorphic forms*: Iwaniec is a leader in the analytic number theory of modular (and automorphic) forms, so the choice of material in these books is more in line with what analytic number theorists are interested (very interesting material that is not really found in most of the other books in this list).

• Apostol, *Modular functions and Dirichlet series in number theory*: another analytic number theory text, but this one is rooted in old-school analytic number theory.

• Ogg, *Modular forms and Dirichlet series*: an older text that consequently provides a different take on some things.