Polar Coordinates

What is a coordinate system?. (Class)

Polar coordinates:

- The location of a point \( P \) is determined by:
  1. A predetermined origin \( O \) and initial ray
     (usually the Cartesian origin and positive \( x \)-axis)
  2. The signed distance \( r \) from \( P \) to the origin
     (Why ‘signed’ distance?)
  3. An angle \( \theta \) from the initial ray to the ray \( \overrightarrow{OP} \)
     (Why ‘an’?)

- Example: The point \( P \) with Cartesian coordinates \((1, \sqrt{3})\)
  has polar coordinates \((r, \theta) = (2, \pi/3)\) (or \((r, \theta) = (-2, -2\pi/3)\) or \((r, \theta) = (2, 7\pi/3)\)…)

- Warning: this lack of uniqueness for the polar representation can makes solving equations tricky.
• Polar $\implies$ Rectangular conversion
  (by picture - class)
  \[ x = r \cos \theta \]
  \[ y = r \sin \theta \]

• Rectangular $\implies$ Polar conversion
  (by picture - class)
  \[ r^2 = x^2 + y^2 \]
  \[ \frac{y}{x} = \tan^{-1} \theta \]

• **Complex Numbers** If this all looks familiar to you, it is because we've already seen all this before: the conversion between a point's rectangular coordinates $(x,y)$ and polar coordinates $(r,\theta)$ are exactly the same as between a complex number $x + yi$ and its polar representation $re^{i\theta}$.

• These formulas can be used to convert an equation from polar to rectangular coordinates and vice versa. For rectangular to polar, it is usually easiest to just replace $x$ and $y$ by $r \cos \theta$ and $r \sin \theta$. For polar to rectangular, it is often easier to rewrite the equation so it includes terms of the form $r \cos \theta$, $r \sin \theta$, and $r^2$. 
• Examples: (details in class)

1. Find equation in rectangular form: \( r = R \)

2. Find equation in polar form: \((x + a)^2 + y^2 = a^2\)

3. Find equation in polar form: \(y = mx + b, \quad b \neq 0\)

4. Find equation in rectangular form: \(r = \sin \theta\)

5. Graph \( r = \theta \)
6. Graph \( r = 1 + \cos \theta \)
7. Graph \( r = \sin(3\theta) \)