# Math 242 Final Exam

1. (3 points) Let $f(x) = \ln(x+1)$. Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same plot.

2. (5 points) Differentiate
$$y = \ln|\sec 7x + \tan 7x|.$$
   Remember to simplify your answer.

3. (2 points) Let $g$ be a function, and let
$$f(x) = g(e^x).$$
   Find $f'$ in terms of $g'$.

4. (5 points) Evaluate the limit
$$\lim_{x \to 0} \frac{e^{3x} - e^{-3x}}{\ln(x+1)}.$$

5. (5 points) Evaluate
$$\int e^x \cos x \, dx.$$

6. (5 points) Evaluate
$$\int \sin^3 x \cos^2 x \, dx.$$

7. (5 points) Evaluate
$$\int_2^3 \frac{x+5}{x^2 + x - 2} \, dx.$$
   Specify if the integral is proper or improper.

8. (5 points) Evaluate
$$\int_0^1 \ln x \, dx.$$
   Specify if the integral is proper or improper.

9. For each series below, determine whether the series is absolutely convergent, conditionally convergent, or divergent. Justify your answers.

   (a) (3 points) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n-1}}$

   (b) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{2^n}$

(c) (3 points) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{\sqrt{n^4+1}}$

10. (3 points) Consider the series
$$\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\frac{1}{48}+\dots$$
If the series converges, find its sum. If the series diverges, explain why.

11. (5 points) Find the radius of convergence of the power series
$$\sum_{n=0}^{\infty}\frac{n^2(x-4)^n}{(n+1)\,3^n}.$$

12. Let $f(x) = x^{1/4}$.
   (a) (4 points) Find the degree 1 Taylor polynomial $T_1(x)$ of $f$ centered at 1.
   (b) (4 points) Find an upper bound for the error in the approximation $f(x) \approx T_1(x)$ when $0.7 < x < 1.3$. (Since you don't have a calculator, leave your answer in exact form.)

13. The direction field for a differential equation is given below:
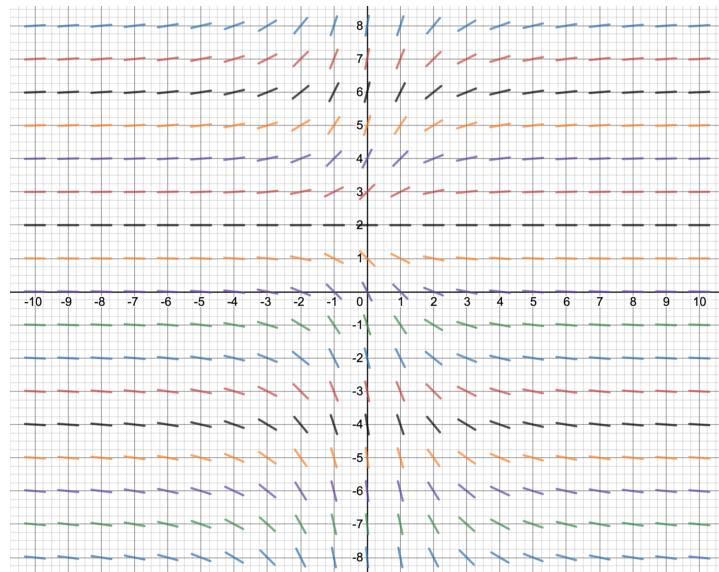


   (a) (2 points) Find all the equilibrium solutions to the differential equation.
   (b) (3 points) Sketch the solution curve to the differential equation that passes through the point $\left(0, \frac{1}{2}\right)$.

14. Solve the following differential equations/initial value problems.
   (a) (5 points) $\cos^2(x)\dfrac{dy}{dx} = 1 + y^2$, $\quad y(0) = \sqrt{3}$.
   (b) (5 points) $xy' + \ln(x)y = \ln(x)$, $\quad x > 1$.

15. (5 points) A tank initially contains 200 L of pure water. Brine that contains 0.5 kg of salt per liter enters the tank at a rate of 8 L/min. The solution is kept thoroughly mixed, and drains from the tank at the same rate. Find the amount of salt in the tank after $t$ minutes.