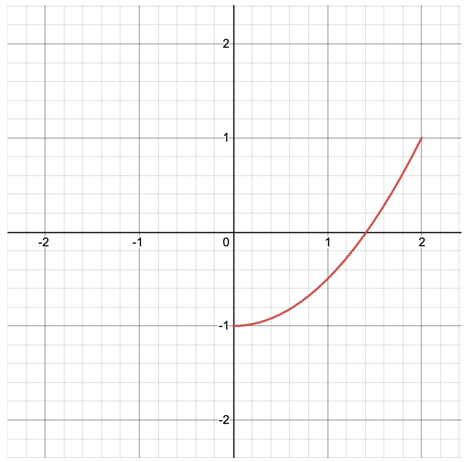
Math 242 Final, Spring 2021

Name:

Question	Points	Score
1	3	
2	4	
3	7	
4	12	
5	13	
6	8	
7	5	
8	5	
9	10	
10	10	
11	32	
12	10	
13	6	
14	7	
15	8	
16	10	
Total:	150	

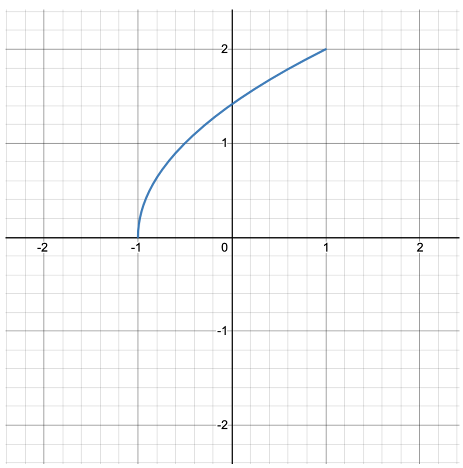
- The exam is open notes and open books, but no calculators are allowed, and no online resources other than the electronic textbook are allowed.
- You can either write your answers on paper and scan or photograph them, or write them on a tablet. You do not have to print out the exam, but can if you like. If you do not print out the exam, make sure you very clearly label which answer corresponds to which problem.
- Make your final answer to each problem clear (for example, by drawing a box around it).
- Make sure you show all your work and justify your answers.
- Please ask if anything seems confusing or ambiguous.
- Good luck!

1. (3 points) Let f be the function with domain $[0, 2]$ and range $[-1, 1]$, with graph as pictured below.

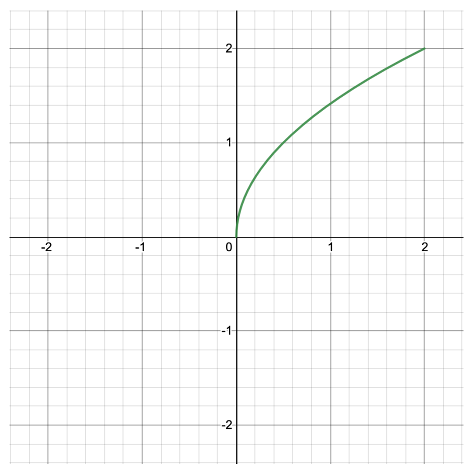


Which of the pictures below represents the graph of f^{-1}?

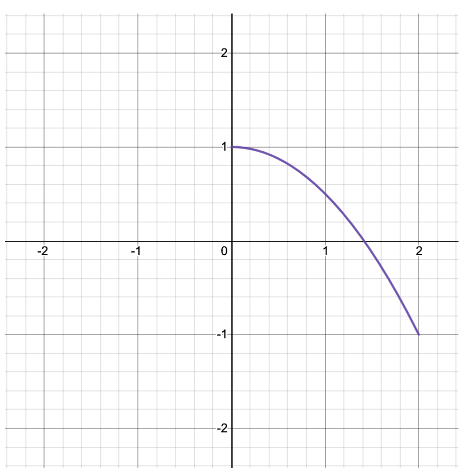
(a)



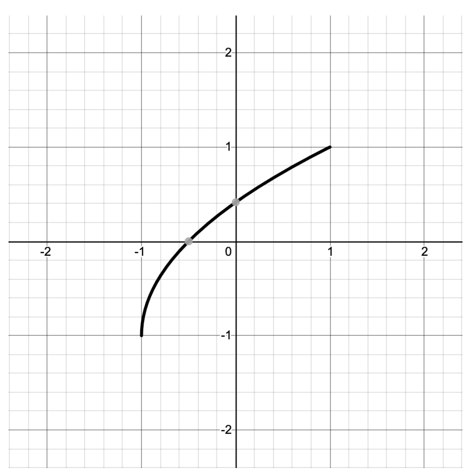
(b)



(c)



(d)



2. (4 points) Say $g(x) = 4x - 7$. Find, and simplify, a formula for $g^{-1}(x)$.

3. (7 points) Recall that *Newton's law of cooling* says that

$$\frac{dT}{dt} = k(T - T_s),$$

where T is the temperature of an object, T_s is the temperature of its surroundings, and k is a constant depending on the physical properties of the object.

A cup of coffee starts at temperature $180°F$ in a room where the surrounding temperature is $80°F$. After 10 minutes, the coffee has cooled to $130°F$. Find an equation for the temperature T (in $°F$) of the coffee as a function of time t (in minutes). Simplify your answer as much as you can, and make sure there are no unknown constants.

4. Compute the following limits. You must justify your solution using algebraic manipulations and / or l'Hôpital's rule for full credit.

   (a) (6 points) $\lim_{x \to 0^+} 2x \ln(x)$.

   (b) (6 points) $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$.

5. (a) (6 points) Use the trapezoidal rule with $n = 4$ to find an approximation to the integral

$$\int_0^4 x^2 dx.$$

(b) (7 points) Using the error formula for the trapezoidal rule, estimate how far your answer to (a) is from the actual value of the integral.

6. (8 points) Compute the integral $\int \frac{2x}{x^2 - 2x + 1} dx$.

7. (5 points) Use an appropriate trigonometric substitution to convert the integral $\int \frac{x^2}{\sqrt{4-x^2}}dx$ into an integral without a square root, in terms of θ (**do not try to solve the integral!**).

8. (5 points) A student uses the substitution $x = 3\tan(\theta)$ to convert an integral to a trigonometric integral, then computes the new integral to get

$$\tan(\theta) + \sec(\theta)\csc(\theta) + C.$$

The problem is not finished as the answer is not in terms of x!

Convert the solution to a function of x that does not involve any trigonometric functions or inverse trigonometric functions.

9. (10 points) Compute the integral $\int_3^\infty xe^{-x}dx$, or say if it diverges. Make sure to justify any limits that are involved.

10. Compute the sum of each of the convergent series below. Simplify your answers.

(a) (5 points) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 3^n}{4^n}$.

(b) (5 points) $\displaystyle\sum_{n=3}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$.

11. For each of the following series, say whether they converge or diverge. For full credit, you must justify your solutions, and state clearly which test(s) you are using (if any).

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{4n+2n^2}$.

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{1+n}$.

(c) (8 points) $\displaystyle\sum_{n=0}^{\infty} \frac{4n^2}{(n+2)!}$.

(d) (8 points) $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n^{7/5}}$.

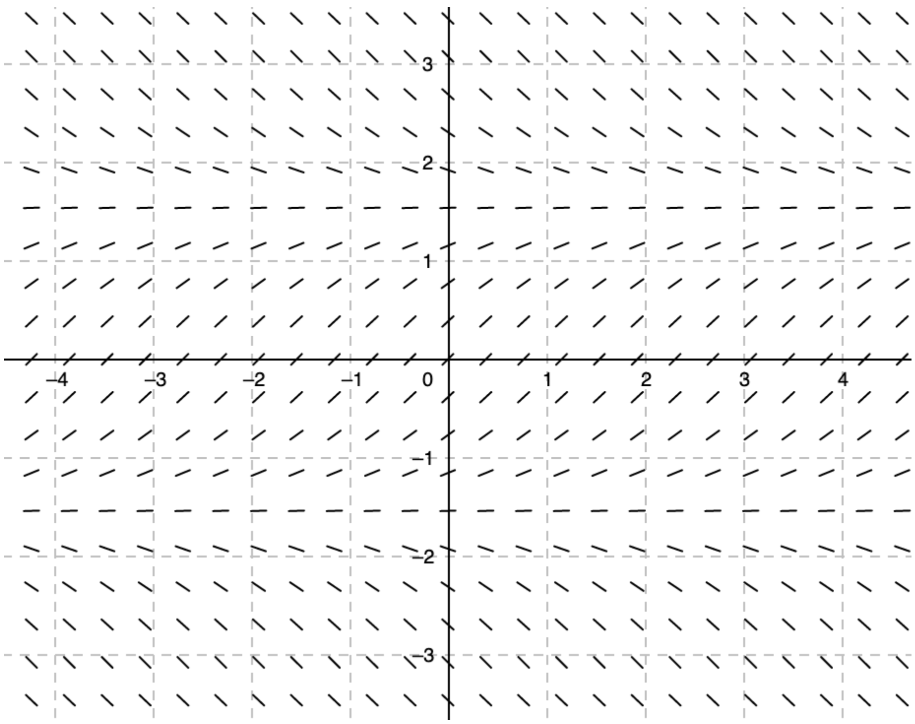
12. (10 points) Find the radius of convergence and interval of convergence of the power series below

$$\sum_{n=1}^{\infty} \frac{2^n}{n^{1/2}} x^n.$$

Make sure you justify your answer.

13. (6 points) Find the order two Taylor polynomial for $\arcsin(x)$, centered at $a = 1/2$.

14. Consider the slope field pictured below.



(a) (2 points) Which of the differential equations below matches this slope field?

(a) $\dfrac{dy}{dx} = \cos(x)$, (b) $\dfrac{dy}{dx} = \sin(x)$, (c) $\dfrac{dy}{dx} = \cos(y)$, (d) $\dfrac{dy}{dx} = \sin(y)$.

(b) (3 points) What are the equilibrium solutions of the ODE?
Hint: use your answer to 1. You can also get partial credit for estimating from the picture.

(c) (2 points) Will a solution that satisfies $y(0) = 0$ be increasing, decreasing, or neither for values of x near 0?

15. (8 points) Solve the differential equation $\dfrac{dy}{dx} = y^2 \cos^2(x) \sin(x)$. Make sure your final answer is (or answers are) expressed as a function (or functions) of x.

16. (10 points) Solve the initial value problem $y' + y = \cos(x)$, $y(0) = 3$. Make sure your final answer is expressed as a function of x.