

SYLLABUS FOR MATH 353
INTRODUCTION TO EUCLIDEAN AND NON-EUCLIDEAN
GEOMETRIES

Course Description: Axiomatic geometry and introduction to the axiomatic method; Euclidean geometry; hyperbolic geometry and other non-Euclidean geometries.

Prerequisite: Math 243 or 253A, and Math 321 (or concurrent); or consent.

Content: This is a survey course on Euclidean and non-Euclidean geometry. Several approaches are possible. Three possible syllabi, each with a slightly different emphasis, are given below. The syllabi are based on the book **Foundations of Geometry**, Second Edition, by Gerard Venema.

Course and program objectives: Many of the students in Math 353 are secondary education majors for whom this is a required course. The way in which this course is taught can have a considerable impact on mathematics taught in high schools.

Students should have a good idea of what it means to implement an axiomatic approach to mathematics. The development of the ability to formulate definitions, present examples and counterexamples, as well the ability to write a direct proof, an indirect proof and a proof by cases should be emphasized.

Possible textbooks:

- John Lee, **Axiomatic Geometry**, AMS.
- Gerard Venema, **Foundations of Geometry**, Pearson.
- Marvin J. Greenberg, **Euclidean and Non-Euclidean Geometry: Development and History**, 4th Edition, W. H. Freeman and Company.
- John Stillwell, **The Four Pillars of Geometry**, Springer.
- Michael Henle, **Modern Geometries: Non-Euclidean, Projective and Discrete**, 2nd Edition, Prentice Hall.
- Patrick J. Ryan, **Euclidean and Non-Euclidean Geometry: An Analytic Approach**, Cambridge University Press.

In the listed syllabi, *Preliminaries* refers to (1) a brief discussion of Euclid and **The Elements**, (2) using the three axioms of incidence as an introduction to axiom systems and the axiomatic method.

All three of the possible syllabi below share the following core topics:

Core topics shared by all three syllabi

<i>Chapter</i>	<i>Topic</i>	<i>Weeks</i>
1 & 2	Preliminaries	≤ 2
3	Axioms	2
4	Neutral Geometry	3
5	Euclidean Geometry	1
7	Area	1

Syllabus 1: Emphasis on Euclidean Geometry

<i>Chapter</i>	<i>Topic</i>	<i>Weeks</i>
1 & 2	Preliminaries	≤ 2
3	Axioms	2
4	Neutral Geometry	3
5	Euclidean Geometry	2
7	Area	1 – 2
8	Circles	1 – 2
10	Transformations	2

Syllabus 2: Emphasis on Non-Euclidean Geometry

<i>Chapter</i>	<i>Topic</i>	<i>Weeks</i>
1 & 2	Preliminaries	≤ 2
3	Axioms	2
4	Neutral Geometry	3
5	Euclidean Geometry	1
6	Hyperbolic Geometry	2
7	Area	1 – 2
11	Models	1 – 2
12	Geometry of Space	1

Syllabus 3: Comprehensive

<i>Chapter</i>	<i>Topic</i>	<i>Weeks</i>
1 & 2	Preliminaries	≤ 2
3	Axioms	2
4	Neutral Geometry	3
5	Euclidean Geometry	1
6	Hyperbolic Geometry	1
7	Area	1
8	Circles	1
10	Transformations	1
11	Models	1
12	Geometry of Space	1