Spring 2016

Math 442 – Vector Analysis (3)

Vector operations, wedge product, differential forms, and smooth mappings. Theorems of Green, Stokes, and Gauss, both classically and in terms of forms. Applications to electromagnetism and mechanics.

Pre: 244 or 253A, and 307 or 311; or consent

I. Differentiable function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^p$. (9 meetings)

We would emphasize the cases $1 \leq n, p \leq 3$. The goal of this section would be to introduce the tangent bundle of $\mathbb{R}^n$, $T\mathbb{R}^n$, and realize the differential of f, df, as a map from $T\mathbb{R}^n \longrightarrow T\mathbb{R}^p$. Diffeomorphisms and change of coordinates would be introduced as well as representations for differentials, tangent vectors, and length of tangent vectors in different coordinates.

II. Vector Fields. (10 meetings)

Vector fields would be treated as dynamical systems as well as differential operators. When viewed as dynamical systems the ideas of integral curve and associated 1–parameter group of diffeomorphisms would be emphasized. When viewed as differential operators we would consider how vector fields act on functions and other vector fields, say, through Lie differentiation or covariant differentiation.

III. Frame Fields. (4 meetings)

Various geometrical and mechanical objects would be studied by means of frame fields. Also covariant derivatives would be studied using frame fields.

IV. Differential Forms and Stokes' Theorem. (11 meetings)

One–forms would be introduced as integrands of line integrals. Then forms, in general, would be studied along with exterior differentiation. Stokes' Theorem would be proved. All this would be reinterpreted in terms of grad, curl, div, and standard vector analysis integral theorems.

V. Applications. (4 meetings)

We could consider the deRham Theorem and introduce deRham cohomology. The Poincaré Index Theorem could be presented. The Laplacian and related Green's formulas could be studied. Also problems in the calculus of variations could be presented. In particular, the problem of finding the shortest curves on surfaces would lead naturally to the differential geometry in Math 443.

Course objectives and Student Learning Outcomes. Upon successful completion of Math 442, the student will have a basic understanding of and will be able to use the key concepts from vector analysis, the meaning of vector and frame fields and differential forms, integral and differential calcultions using these, and the generalized Stokes' Theorem. Students should also understand how these tools are applied.

Program Objectives. Students should gain a basic understanding of the key concepts from vector analysis that are foundational to many areas in mathematics, physics and engineering. In addition, students should further develop their ability to read and write proofs, and with this their understanding of proof as the scientific method of mathematics.