Spring 2016

Math 443 – Differential Geometry (3)

Properties and fundamental geometric invariants of curves and surfaces in space; applications to the physical sciences.

Pre: 244 or 253A, and 311; or consent.

I. Space curves
   A. Definition of space curve
   B. Frenet frames and the formula of Frenet
   C. Curvature and torsion
   D. The Fundamental Theorem of space curves

II. The fundamental tensors of a surface
   A. Definition of a surface
   B. Tangent vectors and differential forms on a surface
   C. The first fundamental tensor
   D. The second fundamental tensor
      1. Normal curvature, principal directions and curvatures
      2. Gauss and mean curvature
   E. Examples and computational techniques

III. Intrinsic geometry of surfaces
   A. The structural equations
   B. Gauss' Theorema Egregium
   C. Covariant derivative
      1. Parallel transport of a vector around a curve
      2. Geodesics
   D. Integration on surface
   E. Gauss–Bonnet theorem

IV. Variational problems
   A. 1st variation of arc length on a surface
   B. 2nd variation of arc length and application
   C. Minimal surfaces

V. Applications
   A. Analytic Mechanics
   B. Elasticity

Course objectives and Student Learning Outcomes. Upon successful completion of Math 443, the student will have a basic understanding of and will be able to use the key concepts from differential geometry, the meaning of metric and curvature, covariant derivative and geodesics, and intrinsic geometry of surfaces. Students will also be introduced to the Calculus of Variations. Students should also understand how these tools are applied.

Program Objectives. Students should gain a basic understanding of the key concepts from differential geometry that are foundational to many areas in mathematics, physics and engineering. In addition, students should further develop their ability to read and write proofs, and with this their understanding of proof as the scientific method of mathematics.