

Math 454 – Axiomatic Set Theory (3)

Sets, relations, ordinal arithmetic, cardinal arithmetic, axiomatic set theory, axiom of choice and the continuum hypothesis.

Pre: Math 321 or graduate standing in a related field or consent.

1. Notation and operations (6 hours)

Includes: Set notations. Notation from and use of predicate logic. Basic set operations (intersection and union, complementation, power set) with corresponding formal axioms from ZF.

2. Relations (5 hours)

Includes: Pairs, relations (including equivalence relations, order relations, well-orders, trees, functions, cartesian products). Remainder of ZF axioms.

3. Induction (3 hours)

Includes: Induction on ω and equivalent forms. Inductive sets. Inductive definitions. Induction on trees and other well-founded relations.

4. Equinumerosity (5 hours)

Includes: Finite sets (equivalent definitions). Countability. Uncountability and Cantor diagonalization. Equinumerosity and Schröder–Bernstein.

5. Ordinals (7 hours)

Includes: Definition of ordinal numbers. Ordinal arithmetic. Transfinite induction and recursion.

6. Cardinals (6 hours)

Includes: Definition of cardinal numbers. Cardinal arithmetic.

7. Axiom of Choice (AC) (4 hours)

Includes: Axiom of choice. Equivalent to AC. Weaker forms. Discussion of independence.

8. Continuum Hypothesis (CH) (2 hours)

Includes: Definition of CH and GCH. Discussion of independence.

9. Miscellaneous (3 hours)

Optional topics and applications (such as number systems; applications of CH; Banach–Tarski paradox; infinitary games; or whatever else the instructor is interested in).

Suggested text: *Elements of Set Theory*, by Herbert Enderton.

Course Objectives:. Upon successful completion, the student will have a foundation in the basic topics of set theory listed above in the syllabus. Emphasis on rigor will provide students the understanding needed in Math 455, in graduate work, and in the study of the logical foundations of mathematics.

Program Objectives:. Set theory is the foundation and language of mathematics. Math 454 introduces the underlying theory in a rigorous way. The course will develop the ability to understand the world from such a point of view and to be able to state mathematical concepts in a precise way.