The Euler Officer Problem

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In 1782 Euler posed the following problem which has come to be known as the EULER OFFICER PROBLEM: 6 officers of the same 6 different ranks are selected from each of 6 regiments. Is it possible to arrange these 36 officers in a 6×6 array so that in each row and column of the array no two officers come from the same regiment or have the same rank? In modern day vernacular:

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

$\longrightarrow$

Can the 36 ordered pairs formed from the numbers 1, 2, 3, 4, 5, and 6 be arranged on a 6×6 board so that in each row and column of the array no two ordered pairs have the same first coordinate or the same second coordinate?

In 1900, 118 years later, Gaston Tarry used brute force to show that the Euler Officer Problem had NO solution! However the work on this problem led to the more general problem of constructing "Euler Squares". An $n \times n$ Euler square is an arrangement of the n^2 ordered pairs formed from the numbers $1, 2, 3, 4, \ldots, n$ on an $n \times n$ board so that no two ordered pairs in the same row or column have the same first coordinate or the same second coordinate.

A huge amount of work has been done on this problem over the years which resulted in 1960 in showing that an $n \times n$ Euler square exists for every n EXCEPT for $n = 2$ (a trivial exercise) and $n = 6$ (= the Euler Officer Problem and a far from trivial exercise). The work on this problem spawned the general area of mathematics known today as "design theory".

This is an ELEMENTARY survey of the history of the solution of constructing Euler squares for all n, except for $n = 2$ and 6, for which no such solution exists.

Key words: Euler squares, orthogonal latin squares, finite fields, block designs.