

# **Schedule for Hawaii Conference in Algebraic Number Theory, Arithmetic Geometry, and Modular Forms, March 6 – 8, 2012**

## **Tuesday, 6 March**

8:00am	Registration opens (Lanai, street level)
8:45 – 9:00am	Welcome, opening remarks (Koi Room, garden level)
9:00 – 10:00am	Plenary address (Bennett) (Koi Room, garden level)
10:10 – 12:10pm	Special Sessions (see supplement)
12:10 – 1:45pm	Lunch
1:45 – 4:20pm	Special Sessions (see supplement)
4:30 – 5:30pm	Plenary address (Lorenzini) (Koi Room, garden level)
5:30 – 8:00pm	Reception (bar closes at 7:30pm) (Wailana Room, garden level)

## **Wednesday, 7 March**

8:30 – 10:30am	Special Sessions (see supplement)
10:40 – 11:40am	Plenary address (Lapidus) (Koi Room, garden level)
12:45 – ???	Excursions

## **Thursday, 8 March**

9:00 – 10:00am	Plenary address (Silverberg) (Koi Room, garden level)
10:15 – 12:00pm	Special Sessions (see supplement)
12:00 – 1:30pm	Lunch
1:30 – 3:45pm	Special Sessions (see supplement)
4:00 – 5:00pm	Plenary address (Ono) (Koi Room, garden level)
5:00pm	Closing remarks (Koi Room, garden level)

## Supplement: Plenary Sessions (held in Koi Room, garden level)

### Bennett:

Title: Generalized Fermat Equations

Abstract: In this talk, we will survey the "state-of-the-art" on generalized Fermat equations of the shape  $x^p + y^q = z^r$ , where  $p$ ,  $q$ , and  $r$  are positive integers, the sum of whose reciprocals is less than 1. We will concentrate on infinite families of exponent triples where the corresponding equations have been shown to have no solutions, via methods based upon the modularity of associated Galois representations.

### Lorenzini:

Title: The index of an algebraic variety.

Abstract: Let  $K$  be a field. Suppose that the algebraic variety is given by the set of common solutions to a system of polynomials in  $n$  variables with coefficients in  $K$ . Given a solution  $P=(a_1, \dots, a_n)$  of this system with coordinates in the algebraic closure of  $K$ , we associate to it an integer called the degree of  $P$ , and defined to be the degree of the extension  $K(a_1, \dots, a_n)$  over  $K$ . When all coordinates  $a_i$  belong to  $K$ ,  $P$  is called a  $K$ -rational point, and its degree is 1. The index of the variety is the greatest common divisor of all possible degrees of points on  $P$ . It is clear that if there exists a  $K$ -rational point on the variety, then the index equals 1. The converse is not true in general. We will survey some recent results on the index.

### Lapidus:

Title: Fractal Strings, Complex Dimensions, and the Spectral Operator: From the Riemann Hypothesis to Universality and Phase Transitions

Abstract: See separate document.

### Silverberg:

Title: Deterministic elliptic curve primality proving for special sequences

Abstract: In joint work with Alexander Abatzoglou, Andrew Sutherland, and Angela Wong, we obtain necessary and sufficient conditions for primality of integers in certain sequences, and give a fast deterministic primality proving algorithm for such integers, using elliptic curves with complex multiplication. This fits in a framework laid out by D. V. Chudnovsky and G. V. Chudnovsky in a 1986 paper, and extends techniques recently employed by B. Gross and by R. Denomme and G. Savin. We discuss a general setting, and some of the underlying number theory.

### Ono:

Title: The legacy of Ramanujan's mock theta functions: Harmonic Maass forms in number theory

Abstract: In his last letter to G. H. Hardy (written from his death bed in 1920), Ramanujan wrote about a new beautiful theory of power series that he refers to as "mock theta functions". This small collection of enigmatic power series puzzled mathematicians for many decades. Then in 2002 Zwegers realized the meaning behind these series; he established that these series are pieces of Maass forms. This understanding has inspired much work. The development of general theorems based on Ramanujan's examples has produced many wonderful theorems on a wide array of subjects such as: L-functions and elliptic curves, Additive number theory (partitions), Donaldson invariants, Representation theory, Explicit class field theory. This talk will be a brief account of this story.

## Schedule Supplement: Arithmetic Geometry Session (held in Asia Room, 2nd floor)

### Tuesday, 6 March

#### 10:10 – 12:10pm      **Special Sessions**

10:10 – 10:50      Achter

Title: The moduli space of cubic surfaces

Abstract: For about a decade, it has been known that the moduli space of complex cubic surfaces is open in a certain arithmetic quotient of the complex 4-ball. I explain this phenomenon by showing that, at least away from the prime 2, the moduli space of stable cubic surfaces can be identified with a certain moduli space of principally polarized abelian fivefolds with endomorphism structure. This answers a recent question of Kudla and Rapoport; part of the proof also requires answering an old question of Deligne.

11:10 – 11:50      Moriwaki

Title : Birational Arakelov Geometry

Abstract: Roughly speaking, the purpose of birational geometry is studies of big linear series. Birational Arakelov geometry is an arithmetic analogue of the above analyses. In this talk, I will give an overview of the recent developments of birational Arakelov geometry.

#### 1:45 – 4:20pm      **Special Sessions**

1:45 – 2:25      Kedlaya

Title: Convergence of solutions of p-adic differential equations

Abstract: Given a nonsingular differential equation whose coefficients are analytic functions on a complex domain, it is a familiar fact that the local series solutions around any point in the domain converge on the largest disc around that point contained in the domain. However, the analogous statement for p-adic analytic functions is patently false, as shown by the example of the exponential function. We will discuss some results on the convergence of local solutions of p-adic differential equations, based on my 2010 book and subsequent developments. In the process, some interesting functions on Berkovich analytic curves will appear.

2:40 – 3:20      Lalin

Title: Distribution of zeta zeroes of Artin--Schreier curves

Abstract: We study the distribution of the zeroes of the zeta functions of the family of Artin-Schreier curves over  $\mathbb{F}_q$  when  $q$  is fixed and the genus goes to infinity. We consider both the global and the mesoscopic regimes, proving that when the genus goes to infinity, the number of zeroes with angles in a prescribed interval of  $[-\pi, \pi]$  has a standard Gaussian distribution (when properly normalized). This is joint work with Alina Bucur, Chantal David, Brooke Feigon, and Kaneenika Sinha.

3:50 – 4:20      Ingram

Title: Variation of canonical heights

Abstract: We may associate, to any polynomial over a number field, a canonical height function, which sheds some light on the dynamics of the polynomial, much as the Neron-Tate height on an abelian variety sheds some light on the group structure. We will present some results on how these height functions vary in families.

## Wednesday, 7 March

### 8:30 – 10:30am Special Sessions

8:55 – 9:35 Pries

Title: Mordell-Weil groups via Artin-Schreier extensions

Abstract: We study the Mordell-Weil groups of Jacobians of curves defined over rational function fields of positive characteristic. Under certain conditions, we show that the rank of the Mordell-Weil group can be arbitrarily large by using Artin-Schreier theory to study the order of vanishing of L-functions. In other situations, we give a formula for the rank in terms of the endomorphism ring of a companion Jacobian. The proof uses an Artin-Schreier variant of Berger's construction of surfaces dominated by a product of curves in towers.

9:45 – 10:25 Okuyama

Title: Convergence of multiplier heights to Lyapunov exponents in non-archimedean dynamics

Abstract: we establish an approximation formula of Lyapunov exponents, and as its special case, the convergence of multiplier height to Lyapunov exponents. We also extend the Manning, Przytycki and DeMarco formula to non-archimedean dynamics. A principal application is a partial affirmative answer to repelling density problem under the positivity of Lyapunov exponents.

## Thursday, 8 March

### 10:15 – 12:00pm Special Sessions

10:15 – 10:55 Baragar

Title: Rational points and automorphisms on K3 surfaces

Abstract: In this talk, we investigate the asymptotic growth of the number of rational points with bounded height in orbits under the action of the group of automorphisms of certain K3 surfaces.

11:15 – 11:55 Kawaguchi

Title: Analytic torsion related to the moduli space of cubic surfaces

Abstract: Borcherds constructed a certain automorphic form on the moduli space of cubic surfaces, which is a complex ball quotient of dimension 4. We show that this automorphic form is realized as an equivariant analytic torsion related to some K3 surfaces with  $Z_3$ -action (with some modification), viewed as a function on the corresponding moduli space of K3 surfaces. This is a joint work with Ken-Ichi Yoshikawa.

### 1:30 – 3:45pm Special Sessions

1:30 – 2:10 Kurlberg

Title: On the dynamical Mordell-Lang conjecture

Abstract: Let  $V$  be a variety, and let  $\phi : V \rightarrow V$  be a morphism. If an (infinite) forward orbit of a point intersects a subvariety  $W$  infinitely many times, what can be said about  $W$ ? The dynamical Mordell-Lang conjecture asserts that this can only happen for "the obvious reason", namely that  $W$  is  $\phi$ -preperiodic. We will give a brief background on the conjecture, and using a  $p$ -adic analytic approach, prove it for certain coordinatewise actions. Assuming certain "random map assumptions", the approach should work for more general maps if the mod  $p$  periodic part of orbit avoids the ramification locus of  $\phi$ . However, in sufficiently high dimensions the approach breaks down due to the periods

being too long. We will discuss this in more detail, and present numerical evidence for the validity of the random map assumption in various dimensions. (The overlap with the AMS special session talk will, apart from some introduction/background material, be minimal.)

2:20 – 3:00                      Yasufuku

Title: Finiteness of Integers in Orbits in Higher-Dimensions

Abstract: Silverman has proved finiteness of integers in each orbit of a morphism on  $P^1$ , as long as the morphism satisfies a geometric condition pertaining to ramification. One can generalize this result to morphisms on  $P^n$ , assuming a deep conjecture in Diophantine geometry by Vojta. In this talk, we will discuss specific maps for which finiteness of integers in orbits can be completely determined without any conjectures, including monomial maps and some low-degree morphisms on  $P^2$ . We will also mention partial results for maps on higher-dimensional projective spaces.

## Schedule Supplement: Number Theory & Fractal Geometry Session (held in Sarimanok Room, 2nd floor)

**Tuesday, 6 March**

**10:10 – 12:10pm      Special Sessions**

**1:45 – 4:20pm      Special Sessions**

**1:45 – 2:15      Van Frankenhuijsen**

Title: Complex Dimensions of Fractal Strings

Abstract: The idea of complex dimensions was conceived by Michel L. Lapidus and developed together with collaborators. Together with the work of Lapidus-Pomerance and Lapidus-Maier, one sees an intimate connection between geometric and spectral oscillations, and between spectral oscillations and the zeros of the Riemann zeta function. Explicit formulas, as developed by Lapidus and van Frankenhuijsen, give the most explicit expression for these connections. The general philosophy for the application to the study of the Riemann zeros is as follows: information about the geometric oscillations of a fractal string allows one to obtain such information for the spectrum by a direct computation. The explicit formula gives then information about the Riemann zeros. We will explain this approach in the example of Cantor strings, the complex dimensions of which lie in a vertical arithmetic progression.

**2:30 – 3:00      Kombrink**

Title: Minkowski content and the dynamical  $\zeta$ -function

Abstract: In this talk a relationship between the parallel volume of a self-conformal fractal and the associated dynamical  $\zeta$ -function will be developed. Properties of the dynamical  $\zeta$ -function, as discovered by Parry, Pollicott and Lalley will be discussed. From these properties we will deduce conclusions on the existence of the Minkowski content

**3:10 – 3:40      Lu**

Title:  $p$ -adic fractal strings, complex dimensions, and zeta functions

Abstract: We develop a geometric theory of  $p$ -adic fractal strings and their complex dimensions. We obtain a closed-form formula for the geometric zeta functions  $\zeta_{L,p}$  of the self-similar strings and show that these zeta functions are rational functions in an appropriate variable. It follows that poles and the zeros of  $\zeta_{L,p}$  are periodically distributed along finitely many vertical lines. Moreover, we present an explicit volume formula for the tubular neighborhood of a  $p$ -adic fractal string  $L_p$ , expressed in terms of the underlying complex dimensions. This formula reveals the oscillations in the geometry of the  $p$ -adic fractal strings. (Joint work with Michel Lapidus and Machiel Van Frankenhuijsen.)

**3:50 – 4:20      Kigami**

Title: Dirichlet forms on  $p$ -adic numbers and associated random walks on infinite trees

Abstract: First, we construct a natural tree associated with  $p$ -adic numbers and give an expression of  $p$ -adic numbers as the collection of ends of the infinite tree. Secondly, we construct a class of jump processes on  $p$ -adic numbers from given pairs of eigenvalues and measures. At the same time, we have concrete expressions of the associated jump kernels and transition densities. Then, we construct intrinsic metrics on  $p$ -adic numbers to obtain estimates of transition densities and jump kernels under some regularity conditions on eigenvalues and measures. Finally, transient random walks on the tree associated with  $p$ -adic numbers are shown to induce a subclass of jump processes discussed in the second part.

**Wednesday, 7 March**

**8:30 – 10:30am      Special Sessions**

8:30 – 9:00      Herichi

Title: On the Apollonian integral packing and some of its geometric and arithmetic properties.

Abstract: In this expository talk, we investigate some of the fundamental known geometric characteristics of the class of bounded primitive Apollonian Integral packings (AIP) as well as some of the recent developments in studying its arithmetic Diophantine properties. We also introduce the class of self-similar tilings as defined and studied in the work of Lapidus, Pearse and Winter and present an open problem related to this particular class of AIP in the light of the theory of complex dimensions in fractal geometry which was developed by M. van Frankenhuysen and Lapidus.

9:10 – 9:40      Ibragimov

Title: Towards Mobius classification of ultrametric spaces.

Abstract: We discuss the problem of classification of ultrametric spaces up to Mobius maps. We begin by reviewing known results on the isometries of ultrametric spaces. We then discuss the concept of normalization of ultrametric spaces. We show that the Mobius maps between ultrametric spaces is related to the isometries between their normalizations.

9:45 – 10:15      Essouabri

Title: Complex dimensions and Euler products.

Abstract: After giving an overview of some techniques of current use for studying analytic continuation of Euler products, I will present some connections with the theory of complex dimensions as developed in the framework of Fractal Geometry by Michel Lapidus and his collaborators.»

## Schedule Supplement: Algebraic Number Theory Session (held in Koi Room, garden level)

### Tuesday, 6 March

#### 10:10 – 12:10pm      Special Sessions

10:10 – 10:30      Kang

Title: Noether's problem and unramified Brauer groups

Abstract: Let  $k$  be any field,  $G$  be a finite group acting on the rational function field  $k(x_g : g \in G)$  by  $h \cdot x_g = x_{hg}$  for any  $h, g \in G$ . Define  $k(G) = k(x_g : g \in G)^G$ . Noether's problem asks whether  $k(G)$  is rational (= purely transcendental) over  $k$ . It is known that, if  $\mathbf{C}(G)$  is rational over  $\mathbf{C}$ , then  $B_0(G) = 0$  where  $B_0(G)$  is the unramified Brauer group of  $\mathbf{C}(G)$  over  $\mathbf{C}$ . Bogomolov showed that, if  $G$  is a  $p$ -group of order  $p^5$ , then  $B_0(G) = 0$ . This result was disproved by Moravec for  $p=3,5,7$  by computer computing. We will give a theoretic proof of the following theorem (i.e. by the traditional bare-hand proof without using computers).

Theorem. Let  $p$  be any odd prime number,  $G$  be a group of order  $p^5$ . Then  $B_0(G) \neq 0$  if and only if  $G$  belongs to the isoclinism family  $\Phi_{10}$  in James's classification of groups with order  $p^5$ .

When  $p=3,5,7$ , groups in the isoclinism family  $\Phi_{10}$  are precisely those groups found by Moravec.

When  $G$  is a group of order  $243$  and  $B_0(G) = 0$ , then  $k(G)$  is rational over  $k$  if  $k$  is a field containing enough roots of unity and  $G$  doesn't belong to the isoclinism family  $\Phi_7$ .

10:40 – 11:00      David

Title: Number of reductions of a global elliptic curves with exactly  $N$  points

Abstract: Let  $E$  be an elliptic curve over  $\mathbf{Q}$ . We consider the problem of counting the number of primes  $p$  for which the reduction of  $E$  modulo  $p$  possesses exactly  $N$  points over the finite field  $\mathbf{F}_p$ . In a previous work in collaboration with E. Smith, we obtained an asymptotic formula for the average over a family of elliptic curves, under some hypothesis concerning the short interval distribution of primes in arithmetic progressions. This average order does not depend only on the size of the integer  $N$ , but on some arithmetic properties of  $N$ , in agreement with the Cohen-Lenstra Heuristics.

We address in this talk the question of finding sharp unconditional upper bounds (including the arithmetic factor predicted by the Cohen-Lenstra Heuristics) for the average. This involves the use of a combinatorial sieve to replace short sums over primes by sums over integers with only "large" prime factors.

This is joint work with Vorropan Chandee, Dimitris Koukoulopoulos and Ethan Smith (CRM, Montr'ea).

11:10 – 11:40      Kani

Title: The ring of modular correspondences

Abstract: In this talk I will discuss the structure of the ring of modular correspondences of the modular curve  $X(N)$  and its relation to the ring of endomorphisms of the Jacobian  $J(N)$  of  $X(N)$ .



11:50 – 12:10 Kaneko

Title: The Ramanujan-Serre differential operators and certain elliptic curves

Abstract: For several congruence subgroups of low levels and their conjugates, we derive differential equations satisfied by the Eisenstein series of weight 4 and relate them to elliptic curves, whose associated new forms of weight 2 constitute the list of Martin and Ono of new forms given by eta-products/quotients.

**1:45 – 4:20pm Special Sessions**

1:45 – 2:05 Togbe

Title: On the Diophantine equation  $x^4 - q^4 = py^r$

Abstract: Let  $x, y, p, q, r$  be nonnegative integers where  $p, q, r$  and  $r$  are distinct primes with  $r \geq 3$ . We consider the Diophantine equation

$$x^4 - q^4 = py^r.$$

If  $r=3$ , the equation has no solutions  $(x, y, p, q)$  with  $\gcd(x, y)=1$ ,  $xy \neq 0$ . But if  $r>3$ , the equation has parameterized solutions. We will give a sketch of the proofs of these results. This is a joint work with F. Luca, A. Banolet, and B. Dupuy.

2:10 – 2:30 Akbary

Title: Lower bounds for power moments of L-functions

Abstract: We derive general results regarding the lower bounds for power moments of certain analytic functions which have Dirichlet series representations on a complex half plane. As corollaries of these results we establish lower bounds of conjectured order of magnitude for power moments of several number theoretical L-functions. This is a joint work with Brandon Fodden.

2:40 – 3:00 Sands

Title L-values and annihilation of the tame kernel in multiquadratic extensions of number fields

Abstract: Let  $F$  be a totally real number field and  $E$  be a composite of finitely many imaginary quadratic extensions of  $F$ . Assume that the Lichtenbaum conjecture holds in  $F$  and each  $E$ . Let  $S$  be a finite set of primes of  $F$  containing the infinite primes and all those which ramify in  $E$ , let  $S_E$  denote the primes of  $E$  lying above those in  $S$ , and let  $\mathcal{O}_E^S$  denote the ring of  $S_E$ -integers of  $E$ . For each character  $\chi$  of the Galois group of  $E$  over  $F$ , we obtain a non-trivial annihilator of the  $S$ -tame kernel  $K_2(\mathcal{O}_E^S)$  from the lead coefficient  $L_S^*(-1, \chi)$  of the Taylor series for the  $S$ -modified Artin  $L$ -function for  $\chi$  at  $s=-1$ .

3:10 – 3:30 Chen

Title: Explicit ramification bounds for division fields of Drinfeld modules II

Abstract: This talk will give the details behind the statements announced in my AMS special session talk. I will describe the methods and proofs of a number of results and applications, obtained with Yoonjin Lee, relating to the problem of explicitly bounding the degree of the different divisor of division fields of Drinfeld modules. In rank 2, this involves a detailed study of the Newton polygon and coefficients of the exponential functions associated to a Drinfeld module, making explicit the work developed by Gardeyn. For general rank, we provide a weaker substitute, and then apply both to the problem of obtaining explicit and partially explicit isogeny theorems for Drinfeld modules.

**Wednesday, 7 March**

**8:30 – 10:30am Special Sessions**

8:30 – 8:50 Komatsu

Title: On the existence of number fields with certain prime conditions

Abstract: Let  $n$  be a positive integer, and  $G$  a subgroup of the  $n$ th symmetric group  $S_n$  containing a permutation of length  $n$ . If  $n$  is not greater than 8 and  $G$  is not the 8th cyclic group  $C_8$ , then there exists a number field  $K$  with degree  $n$  such that every prime divisor of  $n$  remains prime in  $K$  and the Galois group of the Galois closure of  $K$  is isomorphic to  $G$ . We present the numbers of explicit polynomials which define such fields  $K$  and which have given heights.

8:55 – 9:15 Vatsal

Title: Some number theory associated to modular forms on  $SL(2)$

Abstract: This lecture will be a continuation of my talk at the AMS sectional meeting. We will discuss some more or less well-known results on automorphic forms on  $SL(2)$ , and give some arithmetic consequences (which may be less well-known). The main idea is to explain how some familiar questions from number theory may be very naturally formulated in terms of automorphic forms on  $SL(2)$ , rather than the usual group  $GL(2)$ .

9:20 – 9:40 Stewart

Title: Well Spaced integers generated by an infinite set of primes

Abstract: In this talk we discuss an old question of Wintner and its resolution by Tijdeman as well as recent developments due to the speaker and Jeongsoo Kim. We shall prove that there exists an infinite set of prime numbers with the property that the sequence of positive integers made up of primes from the set is well spaced. This is joint work with Jeongsoo Kim.

9:45 – 10:15 Waldschmidt

Title: Some remarks on Diophantine equations and Diophantine approximation

Abstract: The theorems of Thue and Mahler on the finiteness of the set of classes of solutions  $(x, y, \epsilon)$  of any equation  $F(X, Y) = E\epsilon$  (where  $F$  is a binary form with coefficients in a number field  $K$  with at least 3 non-proportional linear factors in the field of algebraic numbers,  $E$  is a finite set of places of  $K$  containing the infinite places, while the values  $x, y, \epsilon$  taken by the unknowns  $X, Y, E$ , are  $S$ -integers in  $K$  for  $x$  and  $y$  and a  $S$ -unit in  $K$  for  $\epsilon$ ) is equivalent to the same statement for the restricted set of equations  $XY(X - Y) = E$ . These statements are also equivalent to the finiteness of the set of solutions of the equation  $E_0 + E_1 = 1$ , where the values taken by the unknown  $E_0$  and  $E_1$  are  $S$ -units in  $K$ . A further equivalent statement is the finiteness of any set of  $S$ -integral points on  $\mathbb{P}^1(K)$  minus three points. This last equivalence can be extended to higher dimension, with the generalized  $S$ -unit equation  $E_0 + \dots + E_n = 1$  on the one hand, and the set of points on  $\mathbb{P}^n(K)$  minus  $n+2$  hyperplanes on the other hand. For  $n \geq 2$ , the finiteness results depend on Schmidt's Subspace Theorem, hence are not effective so far. This is joint work with Claude Levesque.

## Thursday, 8 March

10:15 – 12:00pm Special Sessions

10:15 – 10:35 Katayama

Title: Finite symplectic groups of power order

Abstract: M. Newman, D. Shanks and H. C. Williams have shown that a symplectic group  $Sp(2n, q)$  has a square order if and only if  $n=2$  and  $q=p^2$ , where  $p$  is a NSW prime. In this talk, we shall show there are no symplectic groups of higher power order. The proofs depend on several results on Nagell-

Ljunggren equations and the classical result of Størmer on the diophantine equation  $x^2 + 1 = 2y^n$ .

10:40 – 11:10 Kucera

Title: Eigenspaces of the ideal class group

Abstract: Let  $p$  be an odd prime,  $K$  be a finite abelian extension of  $\mathbb{Q}$  whose degree  $[K:\mathbb{Q}]$  is not divisible by  $p$ , and  $G = \text{Gal}(K/\mathbb{Q})$  be its Galois group. Let  $E$  and  $E_{\text{cyc}}$  be the group of units and of circular units of  $K$ , respectively. Let  $(E/\mathbb{Z})_p$  be the  $p$ -Sylow subgroup of their quotient and  $C_p$  be the  $p$ -Sylow subgroup of the ideal class group of  $K$ . In 1977 G. Gras conjectured that these two modules over the  $p$ -adic group ring  $\mathbb{Z}_p[G]$  have the same Jordan-Hölder series, which means that for each irreducible  $\mathbb{Z}_p$ -valued character  $\chi$  of  $G$  the eigenspaces  $(E/\mathbb{Z})_p^\chi$  and  $C_p^\chi$  have the same cardinality. In the same year R. Greenberg proved that the Main Conjecture of Iwasawa theory implies Gras conjecture, so the conjecture was proven in 1984 when B. Mazur and A. Wiles proved the Main Conjecture. In 1990 V. A. Kolyvagin gave a more comprehensible proof of Gras conjecture by means of Euler systems.

This talk will be devoted to an attempt to make the Euler system machinery work in a "non-semi-simple" situation, i.e. relaxing the assumption that  $p$  does not divide the degree  $[K:\mathbb{Q}]$ . More precisely, assuming the  $p$ -Sylow subgroup  $G_p$  of  $G$  to be nontrivial and cyclic (and some other more technical assumptions) we can show again by slightly modified Kolyvagin-Rubin's method that the eigenspaces  $(E/\mathbb{Z})_p^\chi$ , where we have in mind Sinnott's definition of  $E$ , and  $C_p^\chi$  have the same cardinality for each nontrivial irreducible  $\mathbb{Z}_p$ -valued character  $\chi$  of the quotient  $G/G_p$ . As a consequence, one can get an annihilation statement for  $C_p$  in this case, too. This is a joint work with Cornelius Greither.

11:15 – 11:45 Kolster

Title: On  $\lambda$ -invariants of number fields

Abstract: This is joint work with A. Movahhedi. For an odd prime  $p$  we prove a Riemann-Hurwitz type formula for odd eigenspaces of the standard Iwasawa modules over  $F(\mu_{p^\infty})$ , the field obtained from a totally real number field  $F$  by adjoining all  $p$ -power roots of unity. We use a new approach based on the relationship between eigenspaces and étale cohomology groups over the cyclotomic  $\mathbb{Z}_p$ -extension  $F_\infty$  of  $F$ . The systematic use of étale cohomology greatly simplifies the proof and allows to generalize the classical result of Kida about the minus-eigenspace to all odd eigenspaces.

1:30 – 3:45pm Special Sessions

1:30 – 1:50 Dilcher

Title: A mod  $p^3$  analogue of a theorem of Gauss on binomial coefficients

Abstract: The theorem of Gauss that gives a modulo  $p$  evaluation of a certain central binomial coefficient was extended modulo  $p^2$  by Chowla, Dwork, and Evans. In this talk I present a further extension to a congruence modulo  $p^3$ , with a similar extension of a theorem of Jacobi. This is done by first obtaining congruences to arbitrarily high powers of  $p$  for certain quotients resembling binomial coefficients and related to the  $p$ -adic gamma function. These congruences are of a very simple form and involve Catalan numbers as coefficients. As another consequence we obtain complete  $p$ -adic expansions for certain Jacobi sums. (Joint work with John B. Cosgrave).

1:55 – 2:15

Luca

Title: Balancing with powers of Fibonacci numbers

Abstract: Let  $(F_n)_{n \geq 0}$  be the Fibonacci sequence. In my talk, I will show that the only solution of the Diophantine equation

$$F_1^k + F_2^k + \dots + F_{n-1}^k = F_{n+1}^{\ell} + \dots + F_{n+r}^{\ell}$$

$$F_1^k + F_2^k + \dots + F_{n-1}^k = F_{n+1}^{\ell} + \dots + F_{n+r}^{\ell}$$

$$F_1^k + F_2^k + \dots + F_{n-1}^k = F_{n+1}^{\ell} + \dots + F_{n+r}^{\ell}$$

in positive integers  $(k, \ell, n, r)$  is  $(8, 2, 4, 3)$ . This confirms a conjecture of Behera, Liptai, Panda and Szalay. This is joint work with A. Dujella and S. D. Viçaz Alvarado.

2:20 – 2:40

Friedlander

Title: Weyl sums for quadratic roots

Abstract: We study exponential sums of Weyl type taken over roots of quadratic congruences. We are particularly interested in the situation where the range of summation is small compared to the discriminant of the polynomial. We are then able to give a number of arithmetic applications. This is work which is joint with W. Duke and H. Iwaniec.

2:45 – 3:05

Miyake

Title: How to determine those twists of Hessian elliptic curves which have rational points over the base ground fields

Abstract: Hesse's family of elliptic curves is given by

$$H_{\mu} : U^3 + V^3 + W^3 = 3\mu UVW, \quad (\mu \neq 1).$$

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For the cubic polynomial  $R(t; X) := X^3 + tX + t$ ,  $(t \neq 0, -27/4)$ , take a  $3 \times 3$  matrix

$$\Xi(t) := \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -t & -t & 0 \end{pmatrix}$$

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with entries in  $\mathbb{Q}(t)$  whose characteristic polynomial is  $R(t; X)$ . Furthermore, let  $\mathcal{A}_{\mu, t}$  be the commutative subalgebra of  $\mathcal{M}_3(\mathbb{Q}(\mu, t))$  generated by  $\Xi(t)$ . The twist  $\tilde{H}(\mu, t)$  of  $H_{\mu}$  over the splitting field  $\tilde{K}_{\mu, t}$  of  $R(t; X)$  over  $\mathbb{Q}(\mu, t)$  is defined by the matrix

\$\$

$M_t(x, y, z) := x^3 + yX(t) + zX(t)^2 \in \mathrm{M}_3(\mathbf{Q}(\mu, t, x, y, z))$

\$\$

with indeterminates  $x, y, z$  as

\$\$

$\tilde{H}(\mu, t) : \mathrm{Tr} (M_t(x, y, z)^3) = 3\mu, \mathrm{Det} (M_t(x, y, z))$

\$\$

where  $\mathrm{Tr}$  and  $\mathrm{Det}$  denote the trace and the determinant of square matrices, respectively. Hence a rational point of  $\tilde{H}(\mu, t)$  over the base field  $\mathbf{Q}(\mu, t)$  is represented by a matrix  $M$  in the algebra  $\mathcal{A}_{\mu, t}$  which satisfies the above defining homogeneous equation for  $\tilde{H}(\mu, t)$ . We will exhibit a condition for such an  $M$  to exist.

3:10 – 3:30

Knightly

Title: Averages of Maass form  $L$ -functions

Abstract: Using the relative trace formula, we derive an asymptotic expression for the average (over weight 0 cusp forms of a given level, and weighted appropriately) of Maass form  $L$ -functions inside the critical strip. Particular attention is given to the case of newforms of cubic level, for which we obtain an exact formula for the average. This is joint work with Charles Li.

## Schedule Supplement: Modular Forms Session (held in Pacific Room, 2nd floor)

### Tuesday, 6 March

#### 10:10 – 12:10pm      **Special Sessions**

10:10 – 10:30      Kronholm

Title: New Ramanujan Congruence Properties of the Restricted Partition Function  $p(n, m)$

Abstract:  $p(n, m)$  is the restricted partition function that enumerates the number of partitions of  $n$  into exactly  $m$  parts. Recent and ongoing investigation of Ramanujan congruence properties of  $p(n, m)$  has yielded patterns having a tremendous amount of symmetry and balance. Here are two very brief specific examples:

$$p(2940j, 7) \equiv p(2940j-7, 7) \equiv p(2940j-14, 7) \equiv 0 \pmod{7^2}$$

$$p(300j, 6) \equiv p(300j, 5) \equiv p(300j, 4) \equiv p(300j, 3) \equiv p(300j, 2) \equiv 0 \pmod{5}$$

The aim of this presentation is to discuss data and patterns from this line of inquiry on  $p(n, m)$  that share symmetry with known results but have avoided generalization and remain conjectures.

10:40 – 11:00      Lemke Oliver

Title : Eta-quotients and theta functions

Abstract: The Jacobi Triple Product Identity gives a closed form for many infinite product generating functions that arise naturally in combinatorics and number theory. Of particular interest is its application to Dedekind's eta-function, defined via an infinite product, giving it as a certain kind of infinite sum known as a theta function. Using the theory of modular forms, we classify all eta-quotients that are theta functions.

11:10 – 11:30      Griffin

Title : Integrality Properties of Symmetric Functions in Singular Moduli for Non-holomorphic Modular Forms

Abstract: In his paper *Traces of Singular Moduli*, Zagier studied values of modular functions at imaginary quadratic points, known as singular moduli. He proved that traces of these numbers are Fourier coefficients of certain weight  $3/2$  modular forms. Building on this work, we give conditions under which certain symmetric functions in singular moduli of non-holomorphic modular functions are integral or may given explicit bounds on the denominator.

#### 1:45 – 4:20pm      **Special Sessions**

2:10 - 2:30      Petrow

Title: Transition Mean Values, Eisenstein Series and Non-differentiable functions

Abstract: The average value of the Jacobi symbol in both entries is computed easily when the sum in one entry is much longer than the other. When both sums are of proportional size, it was discovered in 2000 that the leading constant in the asymptotic average varies as "Riemann's nondifferentiable function". In recent work, we have found that the same phenomenon occurs for averages of shifted convolution sums of Hecke eigenvalues of classical modular forms. In both cases we explain the strange behavior in the transition region through multiple Dirichlet series, Eisenstein series and automorphic distributions.

2:40 – 3:20      Pellarin

Title: Deformations of vectorial modular forms and applications

Abstract: These objects are introduced in the theory of modular forms in positive characteristic and their interest lies in the fact that they encode arithmetic properties of a class of  $L$ -series in positive

characteristic. In this talk, we give an overview of this theory and we present some open questions. No analogues of these objects are known in the classical theory in characteristic zero yet.

### **Wednesday, 7 March**

**8:30 – 10:30am**      **Special Sessions**

9:15 – 9:35      Carney / Pitman

Title: Hecke actions on powers of the Dedekind eta-function and vector partitions

Abstract: Half-integer weight Hecke operators and their distinct properties play a major role in the theory surrounding partition numbers and Dedekind's eta-function. Generalizing the work of Ono, here we obtain closed formulas for the Hecke images of all negative powers of the eta-function. These formulas are generated through the use of Faber polynomials. In addition, congruences for a large class of powers of Ramanujan's Delta-function are obtained in a corollary. We further use these results to exhibit a fast calculation for many large partition numbers and vector partition numbers. This work was done jointly with Anastassia Etropolski summer 2011 at Ken Ono's REU program.

9:45 – 10:25      Kane

Title: Locally harmonic Maass forms and rational period functions

Abstract: In this talk, we will define certain functions which satisfy weight  $2-2k$  modularity and are harmonic away from certain geodesics. These functions are connected to the kernel function of Kohnen and Zagier for the Shimura and Shintani lifts (between weights  $k+1/2$  and  $2k$ ) through natural differential operators. Using these locally harmonic Maass forms, we give a new proof of the rationality of the periods of the weight  $2k$  (hyperbolic Poincaré series) cusp forms appearing in Kohnen and Zagier's kernel function. This talk is based on joint work with Kathrin Bringmann and Winfried Kohnen.

### **Thursday, 8 March**

**10:15 – 12:00pm**      **Special Sessions**

11:15 – 11:55      Rouse

Title: Quadratic Forms Representing all Odd Positive Integers

Abstract: We consider the problem of classifying all positive-definite integer-valued quadratic forms that represent all positive odd integers. Kaplansky considered this problem for ternary forms, giving a list of 23 candidates, and proving that 19 of those represent all positive odds. (Jagy later dealt with a 20th candidate.) Assuming that the remaining three forms represent all positive odds, we prove that an

arbitrary, positive-definite quadratic form represents all positive odds if and only if it represents the odd numbers from 1 up to 451. This result is analogous to Bhargava and Hanke's celebrated 290-theorem. In addition, we prove that these three remaining ternaries represent all positive odd integers, assuming the generalized Riemann hypothesis.

This result is made possible by a new analytic method for bounding the cusp constants of integer-valued quaternary quadratic forms  $Q$  with fundamental discriminant. This method is based on the analytic properties of Rankin-Selberg  $L$ -functions, and we use it to prove that if  $Q$  is a quaternary form with fundamental discriminant, the largest locally represented integer  $n$  for which  $Q(\vec{x}) = n$  has no integer solutions is  $O(D^{2+\epsilon})$ .

### **1:30 – 3:45pm      Special Sessions**

2:20 – 3:00      El Guindy

Title: Periods and Supersingularity for Elliptic Curves and Drinfeld Modules

Abstract: In this talk, we recall some results on the connections between the periods in families of elliptic curves and the supersingular curves at a given prime in the same family. Guided by this, we present some new results indicating the presence of a parallel theory for (rank two) Drinfeld modules. We also present some general formulas for (rank  $r$ ) Drinfeld modules. This is joint work with Matt Papanikolas.

3:10 – 3:50      Stark

Title: Another Conjecture of Stark

Abstract: Yes, there is more than one!