A mathematician reads the Corona–Kim paper

“The Contract Disclosure Mandate and Earnings Management under External Scrutiny”

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Bjørn Kjos-Hanssen
bjoernkh@hawaii.edu January 7, 2020

Earnings management

Inspector is scrutinizing firms for earnings management
Earnings management = misreporting of earnings

One firm

The inspector wants to maximize:

$$bm^I(\mathscr{F})s - \frac{w}{2}s^2.$$

Here $m^I(\mathscr{F})$ is the inspector's conjecture about the level of manipulation by the agent, given the publicly available information, but before deciding on the level of scrutiny s.

Two firms

We want to maximize

$$f(s_0,\ldots,s_{N-1}) = b(m_0^I(\mathscr{F})s_0 + m_1^I(\mathscr{F})s_1) - \frac{w}{2}(s_0^2+s_1^2) - \frac{\gamma}{2}(s_0 s_1 + s_1 s_0).$$

We have

$$\frac{\partial}{\partial s_i} f = bm_i^I(\mathscr{F}) - ws_i - \gamma s_{1-i}.$$

Setting the partial derivatives equal to zero,

$$bm_0^I(\mathscr{F}) - ws_0 - \gamma s_1 = 0, \quad bm_1^I(\mathscr{F}) - ws_1 - \gamma s_0 = 0.$$

Matrix formulation

$$\begin{bmatrix} w & \gamma \\ \gamma & w \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = b \begin{bmatrix} m_0^I(\mathscr{F}) \\ m_1^I(\mathscr{F}) \end{bmatrix}$$

For $N = 3$ (three) firms, we replace γ by $\gamma/(N-1)$ and have

$$bm_0^I(\mathscr{F}) - ws_0 - \frac{\gamma}{2}(s_1 + s_2) = 0.$$

$$\begin{bmatrix} w & \gamma/2 & \gamma/2 \\ \gamma/2 & w & \gamma/2 \\ \gamma/2 & \gamma/2 & w \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix} = b \begin{bmatrix} m_0^I(\mathscr{F}) \\ m_1^I(\mathscr{F}) \\ m_2^I(\mathscr{F}) \end{bmatrix}$$

Projections

This can be written in terms of the identity matrix $I = [\delta_{ij}]$, the Hadamard identity $1_\odot = [1]$ and the projection $P = \frac{1}{N}1_\odot$:

$$Cs = bm^I(\mathscr{F})$$

where

$$\begin{aligned} C &= \left(w - \frac{\gamma}{N-1}\right) I + \frac{\gamma}{N-1} 1_\odot \\ &= \left(w - \frac{\gamma}{N-1}\right) I + \frac{\gamma}{N-1} NP \\ &= \left(w - \frac{\gamma}{N-1}\right) (I - P) + \left\{\frac{\gamma N + (N-1)w - \gamma}{N-1}\right\} P \\ &= \left(w - \frac{\gamma}{N-1}\right) (I - P) + \{w + \gamma\} P \end{aligned}$$

Inverses

Lemma

The inverse of $x(I - P) + yP$ *is* $\frac{1}{x}(I - P) + \frac{1}{y}P$.

This is because $P^2 = P$, $P = IP = PI$, $I^2 = I$, and $P(I - P) = 0$.

Applying inverses

We can solve for s as follows: by Lemma 1,

$$\begin{aligned} C^{-1} &= \left\{\left(w - \frac{\gamma}{N-1}\right)\right\}^{-1}(I-P) + \{w+\gamma\}^{-1}P \\ &= \frac{N-1}{\Gamma-\gamma}I + \left(\frac{-N\gamma}{(w+\gamma)(\Gamma-\gamma)}\right)P \end{aligned}$$

and then

$$s = C^{-1}bm^I(\mathscr{F})$$

So

$$s_i = b\left(\frac{1}{(N-1)w-\gamma}\left((N-1)m_i^I(\mathscr{F}) - \frac{\gamma}{w+\gamma}\sum_j m_j^I(\mathscr{F})\right)\right)$$

which agrees with Corona& Kim's formula (6).

Interior solution: positive definiteness

Lemma

For a subspace W,

$$\det(aP_W + bP_{W^\perp}) = a^{\dim W} b^{\dim W^\perp}$$

and hence

$$\det(aP + b(I - P)) = ab^{N-1}$$

This is true because it is true in some basis.

Verifying positive definiteness

For optimization to make sense we need the Hessian $[\partial f_{s_i s_j}]$ to be negative definite, i.e., C to be positive definite, to have a local maximum of f. This occurs iff all the eigenvalues are positive. The characteristic polynomial of C is found as follows:

$$\lambda I - C = \lambda I - (w - \gamma/(N-1))I - \frac{\gamma N}{N-1}P$$

$$= \left(\lambda - w + \frac{\gamma}{N-1}\right)(I - P) + (\lambda - w - \gamma)P$$

and so

$$0 \stackrel{!}{=} \det(\lambda I - C) = (\lambda - w - \gamma) \cdot \left(\lambda - w + \frac{\gamma}{N-1}\right)^{N-1}$$

giving $\lambda = w + \gamma$ and $w - \gamma/(N-1)$.

Eigenvalues

These are both positive if $w > \gamma/(N-1) \geq 0$, in particular if $w > \gamma \geq 0$, $N \geq 2$. If $\gamma < 0$ (which Corona and Kim do contemplate) then $w^2 > \gamma^2$, $w > 0$ is exactly what we need to guarantee an *interior solution*.

More parameters

m_i occurs in:

$$r_i = e_i + m_i$$

Reported earnings = earnings + "a bias"
β_i occurs in: The contract (salary) offered by the principal (i.e., shareholders or board) to the CEO is

$$w_i(r_i) = \alpha_i + \beta_i r_i.$$

V_i is the payoff of the principal who seeks to maximize $E[V_i]$.

$$V_i = e_i - d_P m_i s_i - w_i(r_i)$$

Verifying (11) and (12)

Let $B = [\beta_j]$, $S = [s_j]$, $M = [m_j]$, and $A = C^{-1}$. We have $S = AM$ and $kM + d_A S = B$, so

$$kM + d_A AM = B$$

where

$$A = b\left(\frac{N-1}{(N-1)w-\gamma}(I-P) + \frac{1}{w+\gamma}P\right)$$

and

$$kI + d_A A = kI + d_A b\left(\frac{N-1}{(N-1)w-\gamma}(I-P) + \frac{1}{w+\gamma}P\right)$$

$$= \left(k + d_A b\left(\frac{N-1}{(N-1)w-\gamma}\right)\right)(I-P) + \left(k + d_A b\frac{1}{w+\gamma}\right)P$$

Let $\Gamma = (N-1)w$. Using Lemma 1,

$$(k+d_A A)^{-1} = \frac{1}{\left(k + d_A b\left(\frac{N-1}{(N-1)w-\gamma}\right)\right)}(I-P) + \frac{1}{\left(k + d_A b\frac{1}{w+\gamma}\right)}P$$

$$= \left(\frac{\Gamma - \gamma}{(k(\Gamma-\gamma) + d_A b\,(N-1))}\right) I$$

$$+ \frac{b d_A N\gamma}{(k(w+\gamma) + b d_A)(k(\Gamma-\gamma) + (N-1) b d_A)} P$$

And indeed, Corona and Kim's (11) is

$$m_i = \frac{(k(w+\gamma) + b d_A)((N-1)w - \gamma)\beta_i + b d_A \gamma \sum_j \beta_j}{(k(w+\gamma) + b d_A)(k((N-1)w-\gamma) + (N-1) b d_A)}$$

Solution with $\beta_1 = \cdots = \beta_N$

It turns out that β_i is a function of $\sum_j \beta_j$ and hence all β_i are equal, when optimizing

$$E_P[V_i].$$

We check concavity by

$$\frac{d^2 E_P[V_i]}{d\beta_i^2} < 0.$$

We allow $\gamma < 0$, but concavity is still verified using

$$\frac{dm_i}{d\beta_i} > 0, \quad \frac{ds_i}{d\beta_i} > 0$$

both of which follow from $w^2 > \gamma^2$, $w > 0$.