Obligations in a context

Simplified semantics for the deontic logic of contrary-to-duties

Bjørn Kjos-Hanssen

University of Hawaii at Manoa

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Neither Reagan nor Gorbachev must be told the secret.

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- But if one of them is told the secret, then the other one should be told as well. A *contrary-to-duty obligation.*

Conditional obligation

CTD obligations do not formalize well if $O(B \mid A)$ is taken as $A \to O(B)$ or $O(A \to B)$. A separate dyadic operator $O(B \mid A)$ is needed.
Carmo and Jones (1997, 2002, 2013) base it on a function

$$\mathrm{ob} : 2^W \to 2^{2^W}$$

with

$$\models_\alpha O(A \mid B) \iff \|A\| \in \mathrm{ob}(\|B\|),$$

where W is the set of worlds.

Here we propose a simpler function $F : 2^W \to 2^W$ and

$$\mathrm{ob}(X) = \{Y : Y \supseteq F(X)\}.$$

Because of the idea of a given context as a set of worlds, it seems we cannot further simplify to just a relation on worlds $R \subseteq W \times W$.
An alternative notion is obtained by

$$Y \in \mathrm{ob}(X) \iff Y \cap X = F(X).$$

Carmo and Jones' conditions on ob

5(a) $\mathrm{ob}(X) \neq \emptyset$.

5(b) If $Y \cap X = Z \cap X$ then $Y \in \mathrm{ob}(X)$ iff $Z \in \mathrm{ob}(X)$.

5(c) If $Y \in \mathrm{ob}(X)$ and $Z \in \mathrm{ob}(X)$ then $Y \cap Z \in \mathrm{ob}(X)$.

5(d) If $Y \subseteq X$ and $Y \in \mathrm{ob}(X)$ and $X \subseteq Z$, then $(Z \setminus X) \cup Y \in \mathrm{ob}(Z)$.

5(e) If $Y \subseteq X$ and $Z \in \mathrm{ob}(X)$ and $Y \cap Z \neq \emptyset$, then $Z \in \mathrm{ob}(Y)$.

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5(a) $\mathrm{ob}(X) \neq \emptyset$. We only require this for $X \neq \emptyset$. In any case $\mathrm{ob}(\emptyset)$ is not that interesting.

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5(d) If $Y \subseteq X$ and $Y \in \mathrm{ob}(X)$ and $X \subseteq Z$, then $(Z \setminus X) \cup Y \in \mathrm{ob}(Z)$. This becomes the condition $F(X \cap Y) \supseteq F(X) \cap Y$: our standards of perfection can only be relaxed, not strengthened, when restricting the context. If we take $Y \in \mathrm{ob}(X)$ to mean $Y \cap X = F(X)$ then 5(d) is just wrong.

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5(e) If $Y \subseteq X$ and $Z \in \mathrm{ob}(X)$ and $Y \cap Z \neq \emptyset$, then $Z \in \mathrm{ob}(Y)$. Must be weakened. We use: $F(X \cap Y) = F(X) \cap Y$ whenever $F(X) \cap Y \neq \emptyset$: standards of perfection should only be relaxed when absolutely necessary. However, when $Z \in \mathrm{ob}(X)$ is taken as not $Z \supseteq F(X)$ but $Z \cap X = F(X)$, so Z consists of only ideal worlds, then it is okay.

My main argument

Suppose $W = \{a, b, c, d, e\}$ where a is mandatory: $\{a\} \in \mathrm{ob}(W)$.

- $\{a, b, c\} \in \mathrm{ob}(W)$ by 5(d) since it is "if $\{a, d, e\}$ then $\{a\}$" (which is equivalent to "not d or e"). And then
- $\{a, b, c\} \in \mathrm{ob}(\{b, c, d, e\})$ by 5(e). Similarly,
- $\{a, b, d\} \in \mathrm{ob}(\{b, c, d, e\})$. So
- $\{b, c\} \in \mathrm{ob}(\{b, c, d, e\})$ and $\{b, d\} \in \mathrm{ob}(\{b, c, d, e\})$ by 5(b).

CJ (2002) defined by 5(c$^-$):
If $Y \in \mathrm{ob}(X)$ and $Z \in \mathrm{ob}(X)$ and $X \cap Y \cap Z \neq \emptyset$, then $Y \cap Z \in \mathrm{ob}(X)$.
By 5(c$^-$), $\{b\} \in \mathrm{ob}(\{b, c, d, e\})$. But this was obtained without using any desirability property of b.

More generally, in "general position", if

- $\|A\| \in \mathrm{ob}(W)$, then
- $\|A\| \in \mathrm{ob}(\|A \vee B\|)$ by 5(e). Then
- $\|A \vee \neg(A \vee B)\| = \|A \vee \neg B\| \in \mathrm{ob}(W)$ by 5(d) (slide 6). Then
- $\|A \vee \neg B\| \in \mathrm{ob}(\|\neg A\|)$ by 5(e). So
- $\|\neg B\| \in \mathrm{ob}(\|\neg A\|)$ by 5(b).

But this is absurd, as B was fairly arbitrary.

Prisoner's dilemma

D_i = player i defects.
We adopt player 1's point of view.

$$O(D_1 \wedge \neg D_2)$$

$$O(\neg D_1 \mid D_1 \leftrightarrow D_2).$$

This is another way to try to defeat condition 5(e).
CJ might argue that $D_1 \wedge \neg D_2$ is the "true" obligation, whereas D_1 and $\neg D_2$ are not.
They might argue that the true obligation is to minimize your own number of years in jail. Then, by first-order logic, it follows that $D_1 \wedge \neg D_2$ should hold.

Model for prisoner's dilemma

Let $\mathrm{ob}(X)$ consist of all sets containing the most favorable element of X. That is, we have a valuation $v : W \to \mathbb{N}$ on worlds and we let

$$F(X) = \{u \in X : (\forall x \in X)(v(x) \leq v(u)\}.$$

We can recover an ordering by

$$a \leq b \iff (\forall X)(a \in F_X \to b \in F_X).$$

A weak version of 5(e), 5(e$^-$):

If $Y \subseteq X$ and $Z \in \mathrm{ob}(X)$ and $Y \cap Z \neq \emptyset$, and $\overline{Z} \notin \mathrm{ob}(Y)$, then $Z \in \mathrm{ob}(Y)$.

will help if *the worlds are (strictly) linearly ordered in value*.

Let us consider a model in which they are not. Let's say we want to maximize our profits and minimize the variance in our profits. Then some pairs (μ_1, σ_1^2) are better than others (μ_2, σ_2^2) but for some pairs it is hard to declare a preference. So let's say world a is best overall, b and c are incomparable, and d is worst. Then

- $\{a\} \in \mathrm{ob}(W)$, so
- $\{a\} \in \mathrm{ob}(\{a, b\})$ by 5(e$^-$), so
- $\{a, c, d\} \in \mathrm{ob}(W)$ by 5(d), so
- $\{a, c, d\} \in \mathrm{ob}(\{b, c, d\})$ by 5(e$^-$), since $\{a, c, d\}$ is still possible and in fact not forbidden in this new context.

And so we have a preference for c over b, which is bad.

History of condition 5(e)

- Proposed by Carmo and Jones (published 1997) with the motivation $Y = \mathrm{av}(w)$, the set of actually possible versions of w, and $X = \mathrm{pv}(X)$, the set of potentially or ideally possible versions of X.

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- Carmo and Jones discuss the counterargument and defeat it by weakening condition 5(c) (*Deontic logic and contrary-to-duties*, 2002)
- Completeness results published (2013)
- K-H gives a stronger version of the same counterargument (2016)

Fence scenario

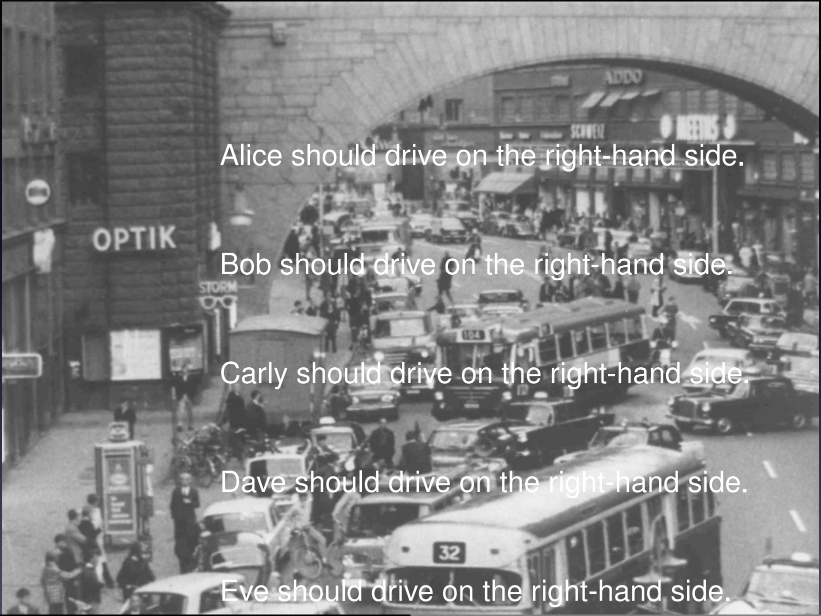
- There ought to be no dog.
- If there is dog, there ought to be a frontyard fence and a backyard fence.
- No point in having a frontyard fence if there is no backyard fence.

may appear to defeat 5(e): let Y be "there is no frontyard fence" and let Z be "there is a backyard fence", and let $X = W$ (no restriction).

Fix Use 5(e$^-$): require $W \setminus Y \notin \mathrm{ob}(X)$ in the antecedent.

CJ No fix is needed! "There ought to be a backyard fence" is not a *true* obligation, only the conjunction of backyard and frontyard is.

Obligation satisfying 5(e)



Alice should drive on the right-hand side.

Bob should drive on the right-hand side.

Carly should drive on the right-hand side.

Dave should drive on the right-hand side.

Eve should drive on the right-hand side.

Traffic scenario

Two cars are driving down the same street in opposite directions.

- A is the proposition that car A is driving on the right side of the street.
- B is the proposition that car B is driving on the right side of the street.

1. There is a primary obligation that $A \leftrightarrow B$. This one is implied by the laws of all countries.
2. Then there is a secondary obligation (which is easier to administrate, and which implies the primary one) that $A \wedge B$. However, some countries instead use $\neg A \wedge \neg B$.

Note however that because of the primary obligation, if $\neg A$ is a fixed fact then $\neg B$ becomes an obligation.

Standard conditional models

We may note that our new $O(\cdot \mid \cdot)$ definition and semantics makes it a normal conditional logic (i.e., extending *CK*), and its models are standard conditional models, in the sense of Chellas 1980, if we define $A \Rightarrow B$ as $O(B \mid A)$.
(In fact they fit a special case of standard conditional models in which the truth of $A \Rightarrow B$ does not depend on the current world.)
Chellas does not consider this option. He considers to define $O(B \mid A)$ as $A \Rightarrow O(B)$, and he considers using *minimal* models (like CJ do) rather than the simpler standard models.

Credits

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