Homework 3 – Math 407

1. Section 2.4: Exercises 2(a), 4(b), 6
2. Section 2.5: Exercises 1(a,b)
3. Section 2.6: 5(a), 6(a), 8(a)
4. Consider the linear system

$$\begin{bmatrix} 3 & -1 & & & \\ -1 & 3 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 3 & -1 \\ & & & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \vdots \\ 1 \\ 2 \end{bmatrix}$$

   (a) If the Jacobi method is applied to this linear system, what is the iteration for the ith entry x_i? Split your answer into three cases: $i = 1$, $i = n$, and $2 \leq i \leq n-1$.

   (b) Write a MATLAB function that carries out K steps of the Jacobi method for this linear system, using the zero vector as an initial guess. Use the following template:

```
function x = jacobi(n,K)
...
for k=1:K
        for i=1:n
                if i==1
                        ...
                elseif i==n
                        ...
                else
                        ...
                end
        end
end
```

   (c) Test your function with $n = 100$ and $K = 10, 20, 30$. Report the errors $\|x^{(K)} - x\|$ you obtain using Matlab's `norm` function. (The exact solution is `x=ones(n,1)`.)

   (d) Repeat parts (b) and (c) using the Gauss-Seidel method in place of the Jacobi method.