Homework 5 – Math 407

Handwritten exercises:

1. Let $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and

$$r_1(x) = x_1^2 + x_2^2 - 3,$$
$$r_2(x) = x_1 e^{x_2} - 4,$$
$$r_3(x) = x_1^3 + x_2 - 2.$$

Approximate the minimizer of $\frac{1}{2}(r_1(x)^2 + r_2(x)^2 + r_3(x)^2)$ by:

(a) Applying one step of the Gauss-Newton method with initial guess $x^{(0)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) Applying one step of the Levenberg-Marquardt method with initial guess $x^{(0)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\lambda = 2$. (Use λI rather than $\lambda \operatorname{diag}(A^T A)$.)

2. Using an initial guess $c_1 = 1$, $c_2 = 0$, apply one step of the Gauss-Newton method to find (approximately) the curve $y = c_1 e^{c_2 t}$ that best fits the data $(0, 1)$, $(1, 2)$, $(2, 6)$ in the least squares sense. On a single plot, draw the curve you obtain and the data points.

3. Using an initial guess $c_1 = 1$, $c_2 = \frac{\pi}{2}$, apply one step of the Gauss-Newton method to find (approximately) the curve $y = c_1 \cos c_2 t$ that best fits the data $(0, 2)$, $(1, 0)$, $(2, -3)$ in the least squares sense. On a single plot, draw the curve you obtain and the data points.

Computer problem:

4. Repeat problem (2) in MATLAB, this time doing as many iterations as needed until $\|Dr(x^{(k)})^T r(x^{(k)})\|$ falls below 10^{-10}. Plot the final curve and data points in MATLAB.