Omega Analysis

University of Hawaii Mathematics Department Distinguished Lecture Series

Risk (Mis) Management and the Financial Crisis

The Impact of the All Too Probable

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Risk (Mis)Management and the Financial Crisis

The Impact of the all too probable

This is Joint Work with Ana Cascon

We are grateful to Bradley Shadwick and William H. Shadwick for many useful discussions.

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Risk Management: What went wrong?

- Forecasts of risk by banks, investors and regulators failed to avoid extreme and even catastrophic loss.
- Clearly there were serious lapses in risk management.
- This has led some to claim that:
  - Statistics is incapable of detecting extreme risk in markets
  - Markets failed to do their job of pricing risk
- These claims are simply wrong.

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Risk Management: What went wrong?

- The wrong tools were used.
- Appropriate statistical analysis of market prices provided warning of both the likelihood and severity of loss in advance of the crisis.
- The necessary tools were available:
  - Extreme Value Theory statistics
  - Expected Shortfall (Conditional Value at Risk) *not* Value at Risk
  - These techniques are well within the capabilities of financial market participants and regulators

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'Risk Management' the Wrong Way

- Value at Risk (VaR) has been one central feature of the failure to manage risk.
- The use of the normal distribution as a model of financial returns has been another.
- Both are in widespread use (and are sanctioned by the Basel Committee on Bank Supervision for the calculation of bank regulatory capital).
- *This is a serious and easily corrected flaw.*

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What is Value at Risk?

- 99% Value at Risk is the answer to the question: *"What is the worst loss we should expect 99 days in 100?"*
- Therefore it is *also* the answer to the question: *"What is the least we should expect to lose 1 day in 100?"*
- In either formulation it omits the *critical question:*
- *"What should we expect to lose on that 1 day in 100?"*

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Expected Shortfall

- 99% Value at Risk is simply the dividing line between what happens 99 days in 100 and 1 day in 100.
- 99% Expected Shortfall answers the question: *"What should we expect to lose 1 day in 100?"*
- 99% *Expected Shortfall* (ES) is the *average* outcome on that 1 day in 100.
- *If you can calculate 99%VaR you can, and should, calculate 99% ES.*

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The Wrong Statistical Model

- Statistical estimates can only be reliable when the tools are appropriate to the data.

- The normal distribution is almost always inappropriate in financial markets.

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Normal Distribution Model ES in the 1929 Crash

30 Sept 1929

Worst loss on Dow Jones Index in previous 250 days: - 4.22%

Normal Distribution Frequency of a worse loss: 1 day in 1461 (5.8 years)

Expected Shortfall on exceeding a loss of 4.22%: -4.59%

On 23 October 1929 the Dow Jones Index dropped by 6.33%

Normal Distribution Frequency of a worse loss: 1 day in 58,000 (232 years)

Expected Shortfall on exceeding a loss of 6.33%: -6.67%

On 28 October the Dow Jones Index lost 13.47%

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The Wrong Statistical Model

- Statistical estimates can only be reliable when the tools are appropriate to the data.
- The normal distribution is almost always inappropriate in financial markets.
- Fat tails, i.e. events too extreme and too frequent to be consistent with normality, are generic in financial data.

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The Right Statistical Model

- Extreme Value Theory (EVT) is the branch of probability & statistics designed to deal with fat tails and extreme events.

- It begins with a wonderful result of Fisher and Tippett published in 1928.

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Extreme Value Theory

Fisher and Tippett studied the distribution of largest (or smallest) independent, identically distributed (i.i.d) random variable from a sample of size n, as n tends to infinity.

If F is the distribution from which the samples are drawn then

$$Prob\{X_{max} < r\} = F(r)^n$$

We want to know this distribution up to the action of the 'location scale' transformation group (the proper affine group on the line).

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Extreme Value Theory

Stability Postulate (Statisticians' Version):
If a limiting distribution G exists, it must be the distribution of its own maxima so for positive integers n there must be a location-scale transformation such that:

$$G^n(x) = G(a_n x + b_n)$$

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Extreme Value Theory

Stability Postulate (Geometers' Version):

$$G^{\lambda}(x) = G(g_{\lambda}x)$$

g_{λ} in the proper Affine Group on the line.

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Fisher and Tippett's 'Three Types'

$$G(x) = e^{-e^{-x}} \qquad \text{on } (-\infty, \infty)$$

$$G(x) = e^{-x^{-\alpha}} \qquad \text{on } [0, \infty) \text{ with } \alpha > 0$$

$$G(x) = e^{-(-x)^{\alpha}} \qquad \text{on } (-\infty, 0] \text{ with } \alpha > 0$$

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Fisher and Tippett's 'Three Types'

$$G(x) = e^{-e^{-x}}$$ Gumbel Distribution

$$G(x) = e^{-x^{-\alpha}}$$ Fréchet Distribution

$$G(x) = e^{-(-x)^{\alpha}}$$ Weibull Distribution

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Gnedenko's Domains of Attraction

Gnedenko (1943) gave necessary and sufficient conditions for the limiting distribution of extremes to be one of the Fisher and Tippett 3 Types. (These conditions describe the 'domain of attaction' of each of the types)

For example, the Fréchet distributions are characterized by:

$$\lim_{n\to\infty} \frac{1 - F(x)}{1 - F(tx)} = t^{\alpha}$$

for all t > 0.

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EVT and Peaks Over Threshold

- If L is a loss threshold, we want to know the limiting distribution of returns below the loss threshold as L tends to minus 100% or minus Infinity.
- Picklands (1975) proves the 'one lamp post' theorem.
- If there's a limiting distribution it approaches a 'Generalized Pareto Distribution' in the domain of attraction of one of Fisher and Tippett's 3 types.

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EVT and Peaks Over Threshold

For example, Picklands' Generalized Pareto distributions on the negative half line given by

$$G(x) = \frac{1}{(1 - \frac{x}{\sigma})^{\lambda}}$$

are easily seen to satisfy Gnedenko's condition for the Fréchet distribution with parameter lambda.

(Exercise: Apply the test to the mirror image distribution H(x)=1-G(-x) to show this.)

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EVT and Peaks Over Threshold

Since there's only one lamp post, that's the one we look under. To model losses in returns distributions, we fit tails of the form

$$G(x) = \frac{1}{(1 - \frac{x}{\sigma})^{\lambda}}$$

If lambda is greater than 1, it counts the number of finite moments. Here's what the fit looks like for Citigroup at the end of 2007, with a Normal distribution tail for comparison.

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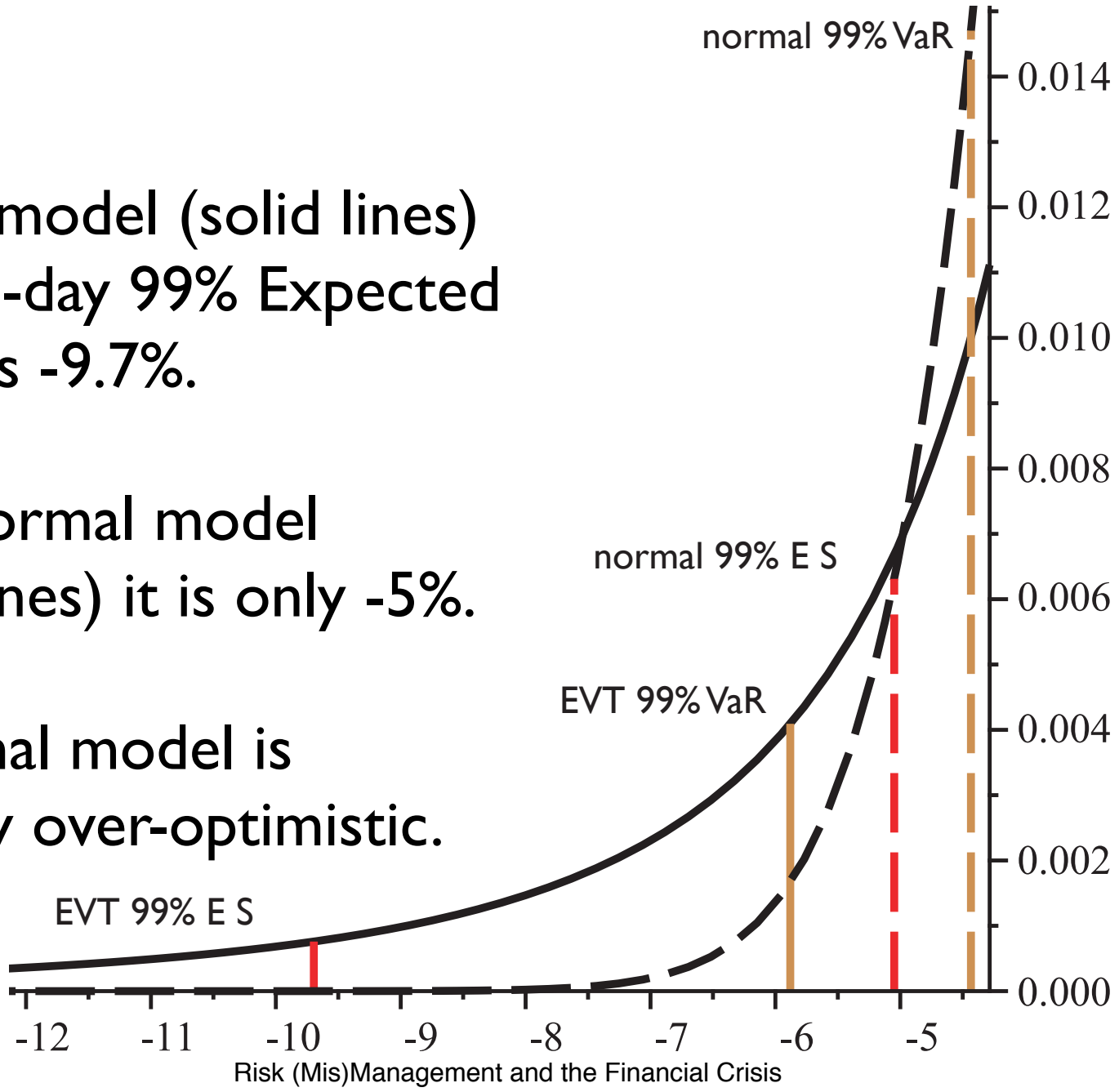
The EVT model (solid lines) says the 1-day 99% Expected Shortfall is -9.7%.

From a normal model (dashed lines) it is only -5%.

The normal model is hopelessly over-optimistic.



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Risk Management the Right Way

- We illustrate what Citigroup management, shareholders and regulators would have seen in the run up to the crisis using appropriate statistical tools.

- The same analysis for major banks in Canada, the EU, the US and the UK (as well as for major market indices) shows that our results are generic.

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Citigroup
What the right statistics had to say.

- Data: Daily return on Citigroup Shares.
  - 250 day rolling data window, i.e. each day the oldest return is discarded and the most recent one added
- Analysis : Omega Metrics® implementation of 'Peaks over Threshold' EVT to fit a Generalised Pareto Tail.
  - 1) Estimate EVT probability of worst loss in the sample and the ES conditional on exceeding this loss
  - 2) Estimate EVT-based 99% VaR and 99% ES to control risk in holding Citigroup shares

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Citigroup
What the right statistics had to say.

- Analysis : Omega Metrics® implementation of 'Peaks over Threshold' EVT to fit a Generalised Pareto Tail.
  - At market close on the last trading day of each month compute EVT probability of worst loss observed in the previous 250 days
  - Estimate the ES conditional on exceeding this loss
  - Compare ES estimate with the average breach of the previous worst loss, if any, over the subsequent month
  - Repeat, updating monthly from January 2007 to April 2009

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Citigroup ES Estimates

Citigroup	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2007	31-Jan-07	-2.47	1 day in 136	-3.73	27-Feb-07	-3.93
	28-Feb-07	-3.93	363	-5.92	-	-
	31-Mar-07	-3.93	278	-5.95	-	-
	30-Apr-07	-3.93	275	-6.02	-	-
	31-May-07	-3.93	277	-6.07	-	-
	30-Jun-07	-3.93	232	-6.22	-	-
	31-Jul-07	-3.93	201	-6.24	9-Aug-07	-5.24
	31-Aug-07	-5.24	241	-8.53	-	-
	30-Sep-07	-5.24	213	-8.52	-	-
	31-Oct-07	-5.24	167	-8.52	1-Nov-07	-6.91
					19-Nov-07	-5.88
					Nov. Average Breach	*-6.39*
	30-Nov-07	-6.91	157	-11.67	-	-
	31-Dec-07	-6.91	148	-11.30	15-Jan-08	-7.28

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Citigroup ES Estimates

Citigroup	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2008	31-Jan-08	-7.28	1 day in 127	-11.94	5-Feb-08	-7.41
	29-Feb-08	-7.41	113	-12.30	-	-
	31-Mar-08	-7.41	79	-12.50	-	-
	30-Apr-08	-7.41	77	-12.12	-	-
	31-May-08	-7.41	70	-11.90	-	-
	30-Jun-08	-7.41	61	-11.81	24-Jul-08	-9.73
					28-Jul-08	-7.56
					Jul. Average Breach	*-8.64*
	31-Jul-08	-9.73	93	-15.57	-	-
	31-Aug-08	-9.73	86	-15.54	15-Sep-08	-15.14
					17-Sep-08	-10.95
					29-Sep-08	-11.89
					Sep. Average Breach	*-12.66*

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Citigroup ES Estimates

Citigroup	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2008	30-Sep-08	-15.14	1 day in 174	-23.91	3-Oct-08	-18.45
	31-Oct-08	-18.45	187	-29.07	19-Nov-08	-23.50
					20-Nov-08	-26.33
					21-Nov-08	-20.00
					Nov. Average Breach	*-23.28*
	30-Nov-08	-26.33	229	-43.00	-	-
	31-Dec-08	-26.33	216	-42.02	-	-
2009	31-Jan-09	-26.33	1 day in 138	-43.32	27-Feb-09	-39.02
	28-Feb-09	-39.02	242	-64.57	-	-
	31-Mar-09	-39.02	210	-64.12	-	-
	30-Apr-09	-39.02	204	-63.31	-	-

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Citigroup
Monitoring Risk With the Right Statistics.

- Analysis : Omega Metrics® implementation of 'Peaks over threshold' EVT to fit a Generalised Pareto Tail.

  – Estimate EVT-based 1-day 99% VaR and 99% Expected Shortfall daily from January 2004 to June 2009 using returns from the previous 250 days

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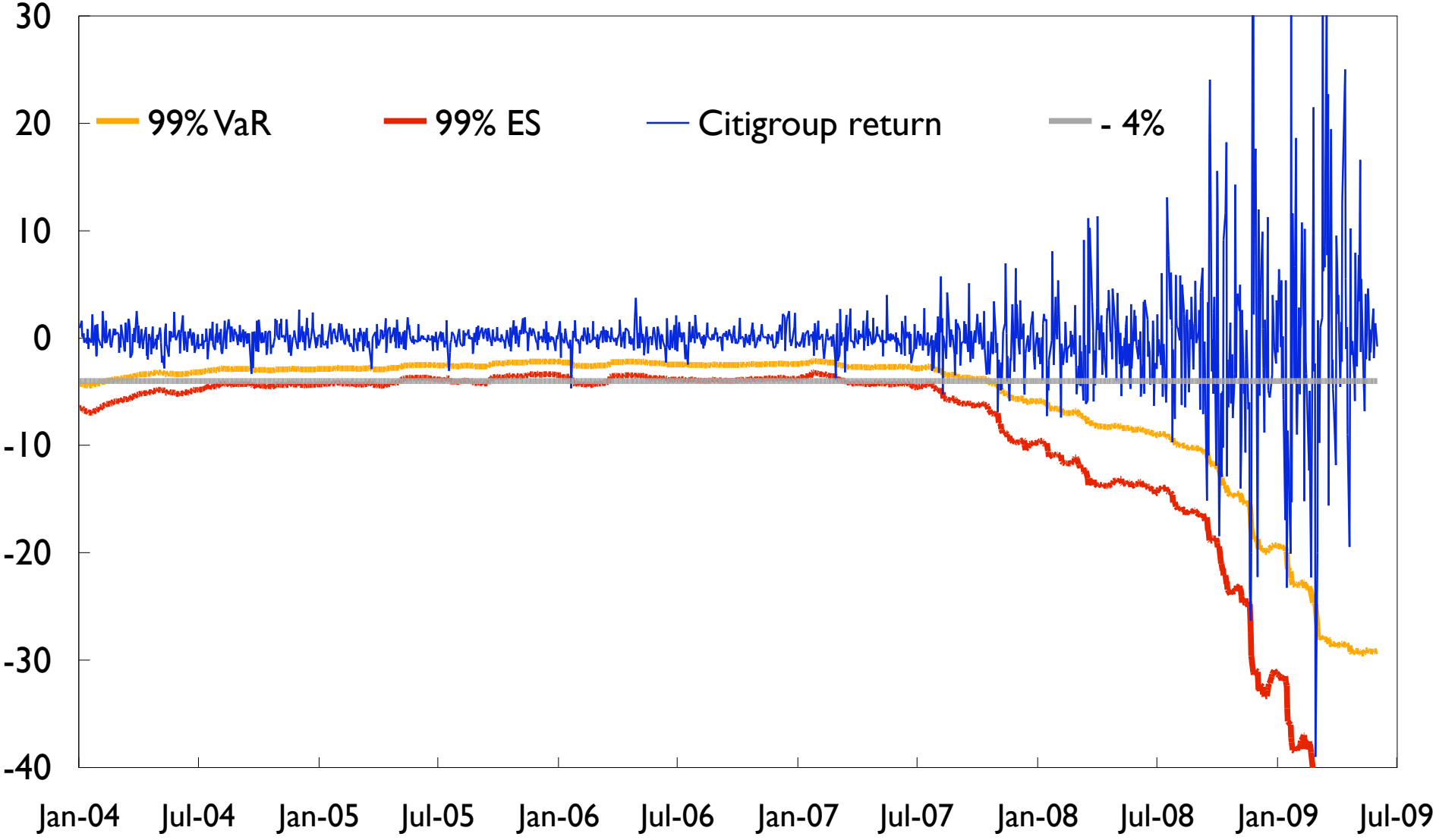
Citigroup EVT 99% VaR and ES



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Citigroup
Controlling Risk With the Right Statistics.

- Analysis : Omega Metrics® implementation of 'Peaks over threshold' EVT to fit a Generalised Pareto Tail.
  - Estimate EVT-based 99% VaR and 99% Expected Shortfall daily from January 2004 to June 2009
  - Construct a risk-controlled portfolio of Citigroup shares and cash, with a target 1-day 99% ES of -4% (No short positions)
  - Compare with the alternative of holding only Citigroup shares with an initial $1million investment

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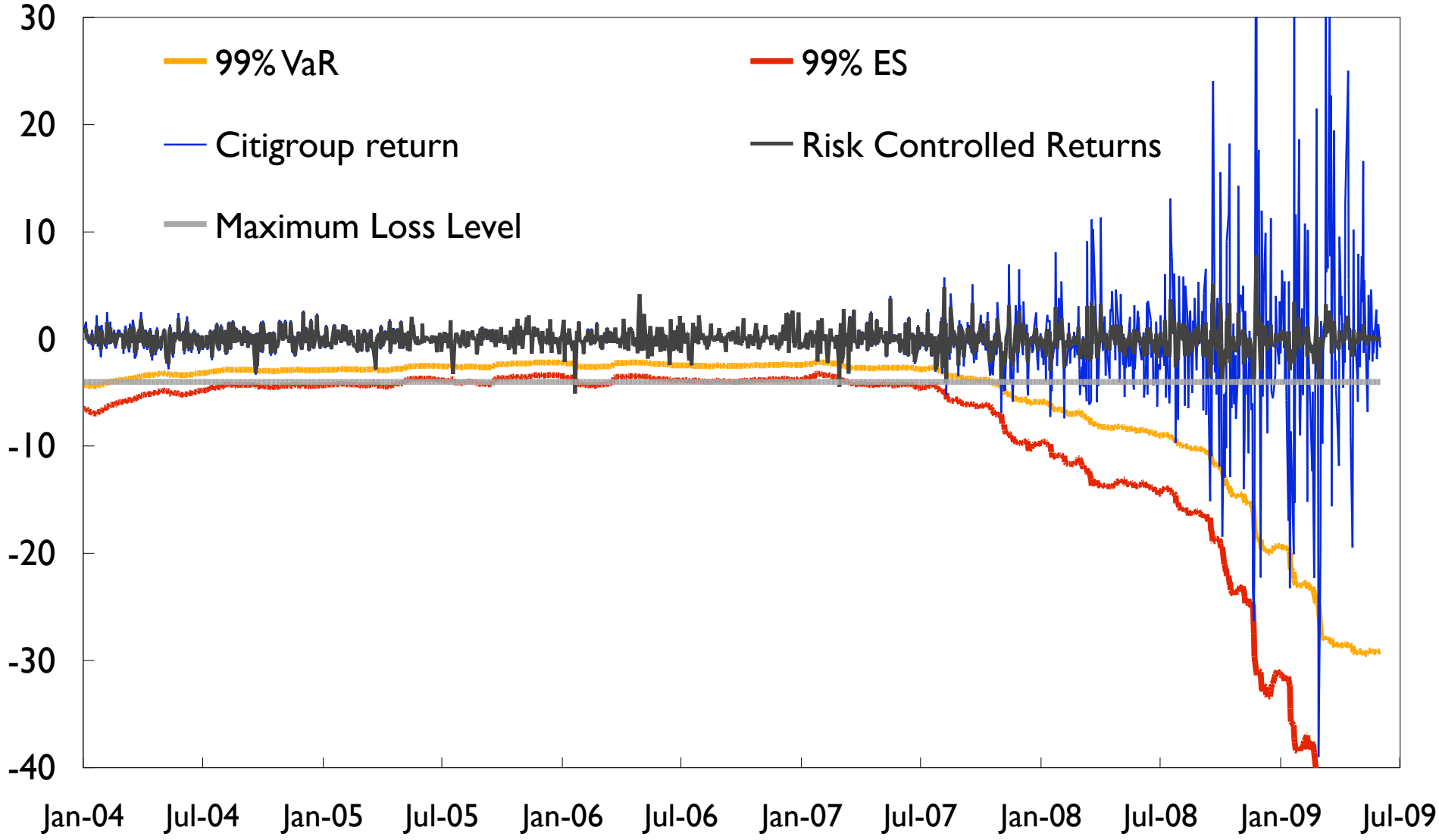
Citigroup EVT Risk Control



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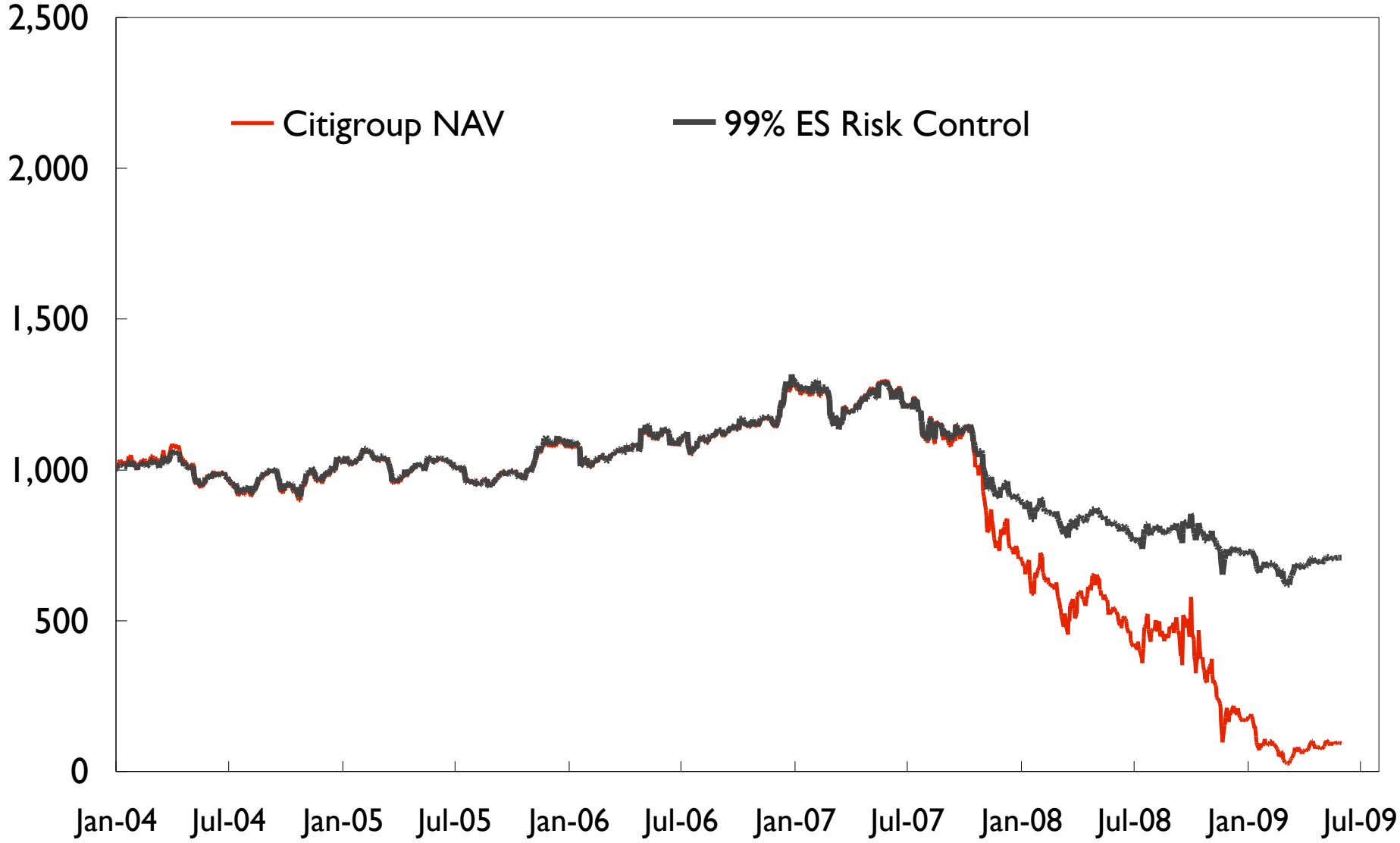
Citigroup NAVs in $1000s



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Citigroup EVT Risk Control

Risk Limit: -4% daily Interest rate: 3% per annum	Citigroup Risk Control	Citigroup Raw
Breaches of -4%	4	97
Average Breach (% per day)	-4.48	-9.11
Worst Loss (% per day)	-5.09	-39.02
Mean Return (% per day)	-0.02	-0.07
Standard Deviation (% per day)	1.10	4.62
Average Gain (% per day)	0.79	2.12
Average Loss (% per day)	-0.77	-2.09
Avg. Gain to Avg. Loss	1.03	1.01
Breaches of EVT 99% VaR	n/a	26
Sample Size	1400	1400

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Dow Jones Index ES Estimates

Dow Jones Index	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2007	31-Dec-06	-1.96	1 day in 110	-3.21	-	-
	31-Jan-07	-1.88	104	-3.10	27-Feb-07	-3.29
	28-Feb-07	-3.29	334	-5.46	-	-
	31-Mar-07	-3.29	285	-5.53	-	-
	30-Apr-07	-3.29	284	-5.53	-	-
	31-May-07	-3.29	329	-5.57	-	-
	30-Jun-07	-3.29	291	-5.77	-	-
	31-Jul-07	-3.29	242	-5.95	-	-
	31-Aug-07	-3.29	160	-6.16	-	-
	30-Sep-07	-3.29	149	-6.17	-	-
	31-Oct-07	-3.29	139	-6.07	-	-
	30-Nov-07	-3.29	103	-6.13	-	-

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Dow Jones Index ES Estimates

Dow Jones Index	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2008	31-Dec-07	-3.29	1 day in 95	-5.89	-	-
	31-Jan-08	-3.29	75	-5.88	-	-
	29-Feb-08	-2.93	58	-5.08	-	-
	31-Mar-08	-2.93	54	-5.01	-	-
	30-Apr-08	-2.93	52	-5.09	-	-
	31-May-08	-2.93	51	-4.90	6-Jun-08	-3.13
					26-Jun-08	-3.03
					Jun. Average Breach	*-3.08*
	30-Jun-08	-3.13	55	-5.12	-	-
	31-Jul-08	-3.13	51	-5.20	-	-
	31-Aug-08	-3.13	55	-5.03	15-Sep-08	-4.42
					17-Sep-08	-4.06
					22-Sep-08	-3.27
					29-Sep-08	-6.98
					Sep. Average Breach	*-4.68*

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Dow Jones Index ES Estimates

Dow Jones Index	Report Date	Worst Return (prev. 250 days)	Probability of Loss	Expected Shortfall Estimate	Breach Date	Breach Return
2008	30-Sep-08	-6.98	1 day in 273	-10.93	9-Oct-08	-7.33
					15-Oct-08	-7.87
					Oct. Average Breach	*-7.60*
	31-Oct-08	-7.87	206	-12.42	-	-
	30-Nov-08	-7.87	136	-12.69	-	-
2009	31-Dec-08	-7.87	113	-13.00	-	-
	31-Jan-09	-7.87	108	-12.97	-	-
	28-Feb-09	-7.87	107	-12.69	-	-
	31-Mar-09	-7.87	99	-12.67	-	-
	30-Apr-09	-7.87	100	-12.46	-	-
	31-May-09	-7.87	100	-12.39	-	-
	30-Jun-09	-7.87	94	-12.85	-	-

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What the right statistics had to say

- This is not special to Citigroup or the Dow Jones Index
- The same analyses produce very similar results for:
  - Lehman Brothers
  - Halifax Bank of Scotland
  - Royal Bank of Scotland
  - BNP Paribas
  - ING
  - Equity Indices (worldwide).
  - Other asset classes
  - Hedge Fund Indices
- Our Analyses are highly efficient
  - Other EVT methods will produce similar results

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What the right statistics have to say.

Additional Results:

- Canadian Banks had significantly less downside going into the crisis than their counterparts in the US, the UK and Europe.
- Price-based triggers for conversion of debt capital instruments for banks and for counter-cyclical regulatory capital
- Evidence for the ability to detect bubbles.

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Risk Management: What Next?

- The solution is *not* a research project: The right tools already exist.
- Statistics didn't fail and Markets didn't fail: Naive statistical analysis of markets failed.
- Careful statistical analysis is the appropriate level of 'mathematical modelling' in finance.

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Risk Management: What went wrong?

- The wrong tools were used. Market Prices contain the necessary information.
- Appropriate statistical analysis would have provided advance warning of both the likelihood and severity of loss in advance of the crisis.
- The necessary tools are available:
  - Extreme Value Theory statistics
  - Expected Shortfall based on EVT
  - These are well within the capabilities of financial market participants and regulators
- They should be adopted.