EXERCISES

problem →

1. This problem (adapted from Mooney & Swift, 1999) refers to Example 7.7. A deterministic model for the yearly population of Florida sandhill cranes is

$$x_{t+1} = (1 + b - d)x_t,$$

where $b = 0.5$ is the birth rate and $d = 0.1$ is the death rate. Initially, the population is 100.

(a) Find a formula for the population after t years and plot the population for the first five years.

Solution →

1. (a) $X_{t+1} = (1 + 0.5 - 0.1)X_t = 1.4X_t$. Therefore the solution is $X_t = 100(1.4)^t$. Your graph will show the population geometrically growing 40% per year. Use Matlab to make the plot.

Problem →

(b) On average, a flood occurs once every 25 years, lowering the birth rate 40% and raising the death rate 25%. Set up a stochastic model and perform 20 simulations over a five year period. Plot the simulations on the same set of axes and compare to the exact solution. Draw a frequency histogram for the number of ending populations for the ranges (bins) 200–250, 250–300, and so on, continuing in steps of 50.

Solution →

(b) In catastrophic years the birth rate is $0.60 \times 0.5 = 0.3$, and the death rate is $1.25 \times 0.1 = 0.125$. At these new rates, $1+b-d = 1.175$, or 17.5% per year. A catastrophic year occurs once every 25 years, or with probability 0.04. To set up a model of the stochatic process we pick a uniform random number in $[0, 1]$ and choose the growth rate to be 1.175 when the random number is less than 0.04, and choose it to be 1.4 otherwise. The following MATLAB m-file performs this task.

```
function sandhill
clear all
X=100; Xhistory=X; T=5;
for t=1:T
c=rand;
if c<=0.04, r=1.175;
else r=1.4;
end
X=r*X; Xhistory=[Xhistory,X];
end
plot(0:T,Xhistory)
```

Now make the histogram.
See help hist for help on setting up the "bins"

problem →

(c) Assume that the birth and death rates are normal random variables,

$$N(0.5, 0.03^2) \quad N(0.1, 0.08^2),$$

respectively. Perform 20 simulations of the population over a five year period and plot them on the same set of axes. ~~On a separate plot, for each year draw side-by-side box plots indicating the population quartiles. Would you say that the population in year 5 is normally distributed?~~

solution →

(c) To simulate a normal random variable $b \sim N(\mu, \sigma^2)$, we use $b = \mu + \sigma z$, where $z \sim N(0,1)$. In MATLAB, the command randn selects a normal RV in [0, 1]. To obtain a simulation as requested, in the previous program replace the time loop by

```
for t=1:T
b=0.5+0.03*randn; d=0.1+0.08*randn;
X=(1+b-d)*X;
Xhistory=[Xhistory,X];
end
```

Skip the part about box plots.

problem →

5. In a generation, suppose thaat each animal in a population produces two offspring with probability $\frac{1}{4}$, one offspring with probability $\frac{1}{2}$, and no offspring with probability $\frac{1}{4}$. Assume that an animal itself does not survive over the generation. Illustrate five realizations of the population history over 200 generations when the initial population is 8, 16, 32, 64, and 128. ← initial pop.
Do your results say anything about extinction of populations?

5. Below is a MATLAB program that produces a time series for the population. The smaller the initial population, the more likely that an extinction will occur at an earlier time.

```
clear all
x=128; xlist=x; sum=0; N=200;
for t=1:N
sum=0;
for j=1:x
r=rand;
if r < 0.25, birth=0;
elseif r > =0.25 & r< 0.75, birth=1;
else r > =0.75, birth=2;
end
sum=sum+birth;
end
xlist=[xlist,sum];
end
T=0:N;
plot(T,xlist)
```

← change initial pop as in problem

For each initial population carry out five simulations