Name (Print):

Math 241 Fall 2018 Exam 1 - Practice 9/26/18 Time Limit: 50 Minutes

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	15	
8	20	
9	15	
Total:	170	

Homework: Recitation: Current Class Grade: 1. (20 points) Find the following limits:

a) 
$$\lim_{x \to 2} \frac{(x^2 - 4)(x + 2)}{(x^2 - 5x + 6)(x + 1)}$$

b) 
$$\lim_{x \to 0} \frac{\tan(2x)}{3x}$$

c) 
$$\lim_{x \to 1} \frac{\sqrt{x+8}-3}{x-1}$$

d) 
$$\lim_{x \to 1^{-}} \frac{\sqrt{3x|x-1|}}{(x-1)}$$

2. (20 points) Find the following limits:

a) 
$$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + 1} + \sqrt{x^2 + x}}$$

b) 
$$\lim_{x \to \infty} \frac{2\sin(x)}{x^2 + 1}$$

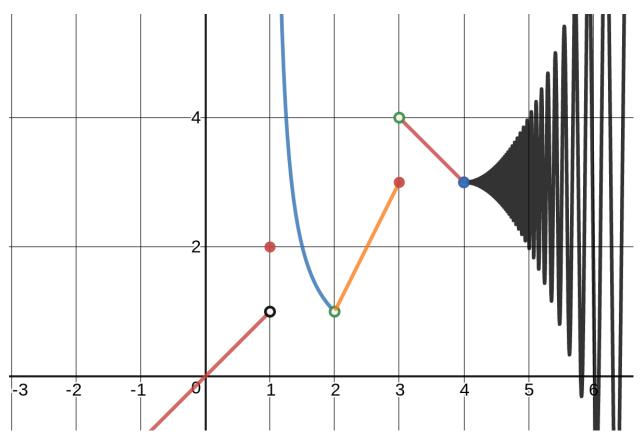
c) 
$$\lim_{x \to 2^-} \frac{x^2 - 10}{x^2 - 4}$$

d) 
$$\lim_{x \to \infty} \frac{(x^{5/3} - 10)\sqrt[3]{x}}{2x^2 + x - 4}$$

3. (20 points) a) State the Intermediate Value Theorem.

b) Use the Intermediate Value Theorem to show that the equation  $\sin(x) + x = 1$  has a solution.

4. (20 points) Below is a graph of y = f(x). For  $0 \le x \le 5$ , state where the following function fails to be continuous and why.



5. (20 points) Use the **definition of the derivative as a limit** to find f'(x) if  $f(x) = \frac{1}{\sqrt{x+1}}$ . Note: using derivative rules will get no points!

- 6. Find the following derivatives: (warning! no partial credit)
  - (a) (5 points)  $f(x) = \sqrt{x} + 7x + \frac{1}{x}$

(b) (5 points)  $g(x) = \tan(x)\sqrt{x^2 + 1}$ 

(c) (5 points) 
$$h(x) = \frac{3 \sec(2x)}{\frac{1}{x^2} + x}$$

(d) (5 points) 
$$k(x) = \sin(1 + \sqrt{x^2 + 2})$$

7. (15 points) Given  $x^2 + y^3 = xy^2 + 1$ , find  $\frac{dy}{dx}$  and give the equation of both the normal and tangent lines at the point (1, 1).

8. (a) (10 points) Given the position function  $p(t) = \sin(t) + \frac{1}{2}t$  (in feet where t is in seconds), find the velocity function v(t), and the acceleration function a(t). On the interval  $[0, 2\pi]$ , when is the object's velocity positive?

(b) (10 points) Given the position function  $p(t) = \frac{1-t^2}{t^2+4}$  (in feet where t is in seconds). Determine when the object is a) At rest

b) Moving forward

c) Moving backward

d) When is the acceleration positive? When is it negative?

e) When is the object speeding up?

9. (15 points) Batman is particular about kites. He ONLY flies a kite at a height of exactly 300 ft, always. Today is no different. Also, today, the wind pushes the kite and it moves horizontally in the air at a rate  $25 \frac{ft}{sec}$ .

a) How fast must he be letting out string for the kite to remain at the constant height of 300 ft. at the precise moment he has let 500 ft. of string out?

b) How fast is the angle between the string and the ground changing at this time?