Name (Print): \_\_\_\_\_\_\_Section:

Problem	Points	Score
1	20	
2	30	
3	30	
4	15	
5	20	
6	10	
7	30	
8	40	
Total:	195	

1. (20 points) Use linearization to approximate  $\sqrt{143}$  and  $\sqrt[3]{124}$ .

2. (a) (10 points) State Rolle's Theorem and the Mean Value Theorem.

(b) (10 points) Use the IVT and Rolle's Theorem (or the Mean Value Theorem) to show that  $2x - \sqrt{2} = \cos^2(x)$  has one, and only one solution.

(c) (10 points) Show that if f is differentiable at x = c, then f is continuous at x = c.

3. (30 points) True of false: If f''(c) = 0 for some c, then f has an inflection point at (c, f(c)).

True of false: If f is differentiable on (a,b) and continuous of [a,b] then  $f'(c) = \frac{f(b) - f(a)}{b-a}$  for any c in (a,b).

True of false: If f'(x) = 0 then f is a constant function.

True of false: If f'(c) = 0 for some c, then x = c is a critical number for f at x = c.

True of false: If f'(c) = 0 for some c, then f has a local min or max at x = c.

4. (15 points) a) Find the absolute extrema of  $f(x) = x^{2/3}(x-6)$  on the interval [-1, 5].

b) Find the absolute extrema of  $f(x) = (x-3)^{2/3}$  on the interval [2, 11].

c) Find the absolute extrema of  $f(x) = \frac{x^3}{3} - 2x^2 + 3x$  on the interval [0, 4].

- 5. (20 points) Consider the function  $f(x) = \frac{x^2 1}{x^2 + 1}$ .
  - a) Determine the interval(s) where f(x) is positive/negative.
  - b) Find  $\lim_{x\to\infty} f(x)$  and  $\lim_{x\to-\infty} f(x)$ . Give the equations of any asymptotes (horizontal, vertical or slant).
  - c) Give the intervals of increase and decrease and give the coordinates of any local min/max (meaning the x and y values).

d) Find the intervals of concavity and the coordinates of any inflection points.

6. (10 points) Sketch a graph of the function from the previous page. Label the asymptotes, extrema and inflection points.

7. (30 points) Repeat the process of problems 5 and 6 for the functions  $f(x) = \frac{x^2}{\sqrt{x+1}}$ ,

$$g(x) = \frac{2x^2}{x^2 - 1}$$
 and  $h(x) = \frac{x^2 + 3}{x - 1}$ .

8. (40 points) a) Find the radius and height of the largest right circular cylinder that can fit inside a sphere of radius 2.

b) What point on the graph of  $f(x) = \frac{1}{\sqrt{x}}$  is closest to the origin?

c) You are to design a, quite large, square bottom box with total volume of 1500 ft.<sup>3</sup>. The mysterious material you are to use costs 2 dollars per ft.<sup>2</sup> and you need to use two sheets of mysterious material on the bottom (this makes the box stronger). Find the dimensions and cost of the cheapest box you can make.