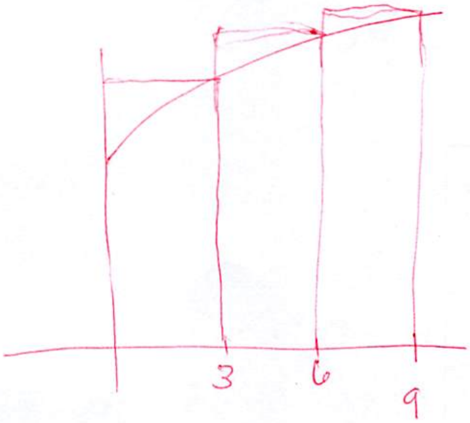
Math 241
Fall 2018
Exam 3 - Practice
11/16/18
Time Limit: 50 Minutes

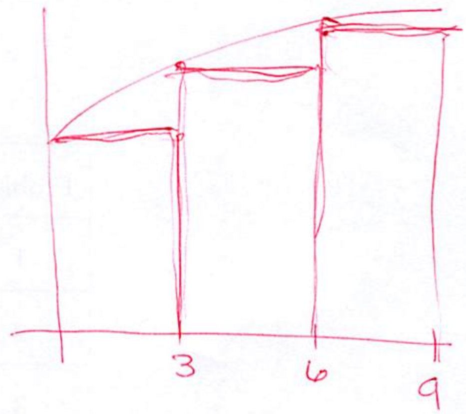
Name (Print): Solutions

Problem	Points	Score
1	20	
2	20	
3	20	
4	80	
5	15	
6	15	
7	20	
8	20	
9	20	
Total:	230	

1. (20 points) a) In the space below, draw a reasonable sketch of $f(x) = \sqrt{x} + 1$ twice (so like two separate graphs that are the same). Then draw 3 equal width rectangles associated to right hand (Riemann) sum approximation over the interval $[0, 9]$ on one of the graphs, and draw 3 equal width rectangles associated to left hand (Riemann) sum approximation over the interval $[0, 9]$ on the other graph.



RHA



LHA

b) Express the areas above in sigma notation.

RHA: $\sum_{k=1}^{3} (\cancel{} + 1) \cdot 3$

$\sqrt{3K}$

LHA: $\sum_{k=0}^{2} (\sqrt{3k} + 1) \cdot 3$

OR $\sum_{k=1}^{3} (\sqrt{3(k-1)} + 1) \cdot 3$

c) For both approximations, determine if they are an overestimate or an underestimate (you don't need to compute anything here, just make you decision based on your drawings in part a).

RHA is over, LHA is under.

2. (20 points) Suppose that a particle has an acceleration function $a(t) = 12t^2 + 2$. If the velocity function, $v(t)$ has the property that $v(1) = 1$, and the position function, $p(t)$, has the property that $p(1) = 0$, find explicit formulas for $v(t)$ and $p(t)$.

$$V(t) = \int a(t)\,dt = 4t^3 + 2t + C_1$$

$$1 = V(1) = 4(1)^3 + 2(1) + C_1$$

so, $C_1 = 1 - 6 = -5$

and $V(t) = 4t^3 + 2t - 5.$

$$p(t) = \int v(t)\,dt$$

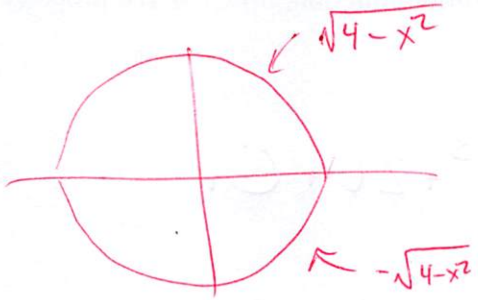
$$= t^4 + t^2 - 5t + C_2$$

$$0 = p(1) = 1^4 + 1^2 - 5(1) + C_2$$

$$\Rightarrow C_2 = 3$$

$$\therefore p(t) = t^4 + t^2 - 5t + 3$$

3. (a) (10 points) Set up a definite integral which gives the area of a circle with radius 2.



$$\int_{-2}^{2} \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right) dx$$

(b) (10 points) Let $G(x) = \int_1^x 1 - t^2 \, dt$. Determine the intervals of increase/decrease and concavity for G.

$G'(x) = 1 - x^2$ so

$G(x)$ is increasing on $(-1,1)$

and decreasing on $(-\infty,-1) \cup (1,\infty)$

$G''(x) = -2x$ so G is C.U. on $(-\infty,0)$

and C.D. on $(0,\infty)$

4. (a) (10 points) $\int \frac{2}{\sqrt[3]{x}} + 5x\ dx$ $= \int 2x^{-1/3} + 5x\ dx$

$= \frac{2x^{2/3}}{2/3} + 5\frac{x^2}{2} + C = 3x^{2/3} + \frac{5}{2}x^2 + C$

(b) (10 points) $\int \frac{1}{\sqrt{x}} + \cos(2x+1) + 2\ dx$ $= \int x^{-1/2} + \cos(2x+1) + 2\ dx$

$= \frac{x^{1/2}}{1/2} + \frac{\sin(2x+1)}{2} + 2x + C$

(c) (10 points) $\int \left(\frac{1}{x^2} + 1\right)^2 dx$ $= \int \frac{1}{x^4} + \frac{2}{x^2} + 1\ dx$

$= \int x^{-4} + 2x^{-2} + 1\ dx = \frac{x^{-3}}{-3} + 2\frac{x^{-1}}{-1} + x + C$

(d) (10 points) $\int_1^2 \frac{x+1}{(x^2+2x)^3}\ dx$ $u = x^2 + 2x$, $du = 2x+2\ dx = 2(x+1)\ dx$ $\Rightarrow \frac{1}{2}du = x+1\ dx$

$\frac{1}{2}\int_3^8 \frac{1}{u^3}\ du = \frac{1}{2}\int_3^8 u^{-3}du = \frac{1}{2}\frac{u^{-2}}{-2}\Big|_3^8 = \frac{-1}{4}\left(\frac{1}{64} - \frac{1}{9}\right)$

(e) (10 points) $\int_0^{1/2} 7\sqrt{\sin(\pi x)}\cos(\pi x)\ dx$ $u = \sin(\pi x)$, $du = \cos(\pi x)\cdot\pi\ dx$

$\frac{1}{\pi}du = \cos(\pi x)\ dx$

$\frac{7}{\pi}\int_0^1 \sqrt{u}\ du = \frac{7}{\pi}\left(\frac{u^{3/2}}{3/2}\Big|_0^1\right) = \frac{14}{3\pi}\left(1^{3/2} - 0^{3/2}\right) = \frac{14}{3\pi}$

(f) (10 points) $\int_0^1 (x+1)^{50}x\ dx$ $u = x+1$, $du = dx$ $\underline{u - 1 = x}$

$\int_1^2 u^{50}(u-1)\ du = \int_1^2 u^{51} - u^{50}\ du = \frac{u^{52}}{52} - \frac{u^{51}}{51}\Big|_1^2$

$= \frac{2^{52}}{52} - \frac{2^{51}}{51} - \left(\frac{1}{52} - \frac{1}{51}\right)$

(g) (10 points) $\int \frac{x^2+2x+1}{x^{3/2}}\ dx$

$= \int x^{1/2} + 2x^{-1/2} + x^{-3/2}\ dx$

$= \frac{x^{3/2}}{3/2} + 2\frac{x^{1/2}}{1/2} + \frac{x^{-1/2}}{-1/2} + C = \frac{2}{3}x^{3/2} + 4x^{1/2} - 2x^{-1/2} + C$

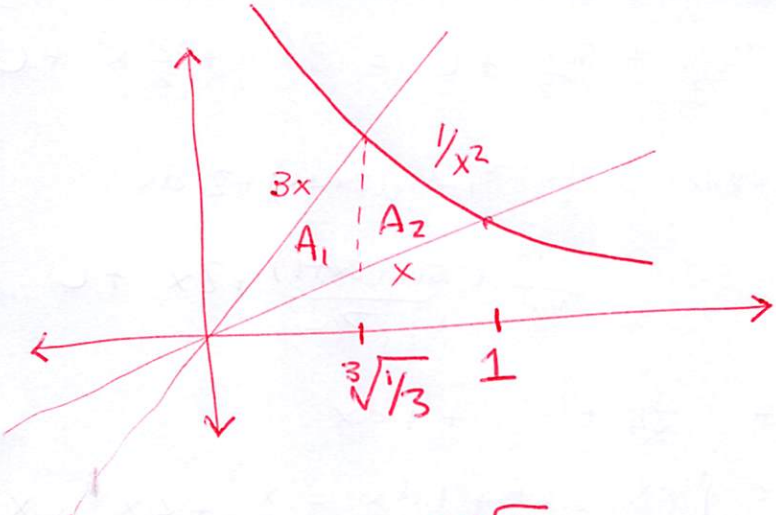
(h) (10 points) $\int 3\sec^2(\sin^2(x))\sin(x)\cos(x)\ dx$

$u = \sin^2(x)$, $du = 2\sin(x)\cdot\cos(x)\ dx$

$\frac{3}{2}\int \sec^2(u)\ du = \frac{3}{2}(\tan(u)) + C$

$= \frac{3}{2}\tan(\sin^2(x)) + C$

5. (15 points) Find the area of the region bounded by $f(x) = \frac{1}{x^2}$, $g(x) = 3x$ and $h(x) = x$.



$$3x = \frac{1}{x^2}$$

$$x^3 = \frac{1}{3} \iff x = \sqrt[3]{\frac{1}{3}}$$

$$A_1 = \int_0^{\sqrt[3]{1/3}} 3x - x\,dx = \int_0^{\sqrt[3]{1/3}} 2x\,dx$$

$$= x^2 \Big|_0^{\sqrt[3]{1/3}} = \sqrt[3]{\frac{1}{9}}$$

$$A_2 = \int_{\sqrt[3]{1/3}}^{1} \frac{1}{x^2} - x\,dx$$

$$= \frac{-1}{x} - \frac{x^2}{2} \Big|_{\sqrt[3]{1/3}}^{1}$$

$$= \frac{-1}{1} - \frac{1^2}{2} - \left(\frac{-1}{\sqrt[3]{1/3}} - \frac{\sqrt[3]{1/9}}{2} \right) = \frac{-3}{2} + \sqrt[3]{3} + \frac{\sqrt[3]{1/9}}{2}$$

So, $A_1 + A_2 = \frac{3}{2} \cdot \sqrt[3]{1/9} + \sqrt[3]{3} - 3/2$

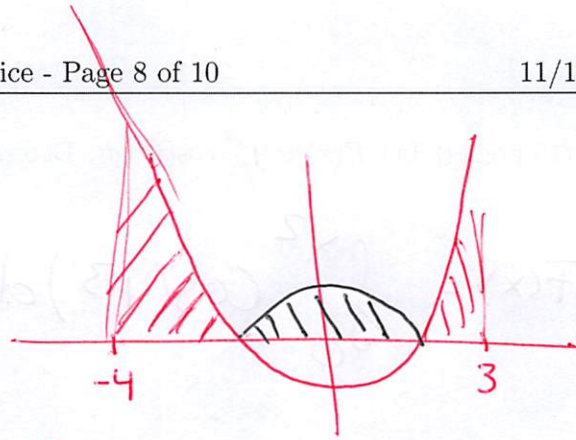
6. (15 points) Let $F(x) = \int_x^{x^2} \cos(t^3)\, dt$. Determine $F'(x)$.

$$F(x) = \int_0^{x^2} \cos(t^3)\,dt - \int_0^{x} \cos(t^3)\,dt$$

So

$$F'(x) = \cos(x^6)\cdot 2x - \cos(x^3)$$

7. (a) (10 points) Find $\int_{-4}^{3} |x^2 - 1|\, dx$.



$$= \int_{-4}^{-1} x^2 - 1\, dx + \int_{-1}^{1} 1 - x^2 dx + \int_{1}^{3} x^2 - 1$$

$$= \frac{x^3}{3} - x \Big|_{-4}^{-1} + x - \frac{x^3}{3} \Big|_{-1}^{1} + \frac{x^3}{3} - x \Big|_{1}^{3}$$

$$= \frac{(-1)^3}{3} - (-1) - \left(\frac{(-4)^3}{3} - (-4)\right) + 1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3}\right) + \frac{3^3}{3} - 3 - \left(\frac{1^3}{3} - 1\right)$$

$$= \frac{-1}{3} + 1 + \frac{64}{3} - 4 + \frac{4}{3} + 6 + \frac{2}{3} = \frac{8}{3} + 2 + \frac{64}{3} = \frac{78}{3} = 26$$

(b) (10 points) Find $\int \frac{\sqrt{\sqrt{\sin(x)} + 1}\cos(x)}{\sqrt{\sin(x)}}\, dx$

$$u = \sqrt{\sin(x)} + 1, \quad du = \frac{1}{2\sqrt{\sin(x)}} \cdot \cos(x)\, dx$$

$$2\int \sqrt{u}\, du = 2\frac{u^{3/2}}{3/2} + C$$

$$= \frac{4}{3}\left(\sqrt{\sin(x)} + 1\right)^{3/2} + C$$

8. (20 points) a) Give all values of b such that $\int_1^b 3x^2 - 3\ dx = 0$.

First, we see that $b=1$ is a solution.

Now, $\int_1^b 3x^2 - 3\,dx = x^3 - 3x \Big|_1^b = b^3 - 3b - (1^3 - 3(1)) = b^3 - 3b + 2$

$b=1$ is a solution so $b-1$ is a factor of b^3-3b+2.

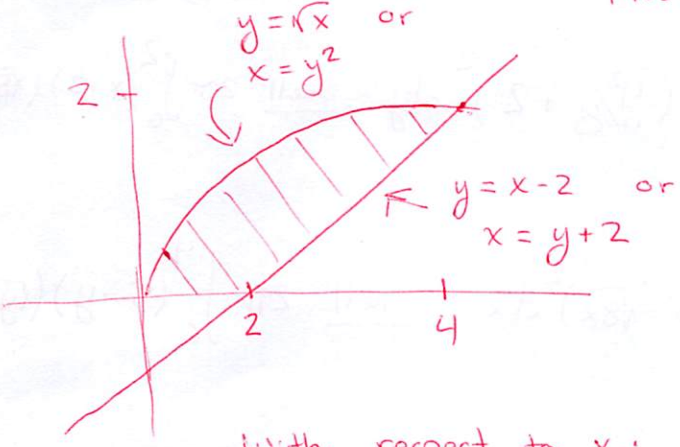
$$\begin{array}{r} b^2+b-2 \\ b-1 \overline{)\, b^3-3b+2} \\ -(b^3-b^2) \\ \hline b^2-3b+2 \\ -(b^2-b \\ \hline -2b+2 \\ -2b+2 \\ \hline 0 \end{array}$$

So, $(b-1)(b^2+b-2) = b^3-3b+2$!

Now, $0 = b^3 - 3b + 2 = (b-1)(b^2+b-2)$
$= (b-1)(b-1)(b+2)$

So, $b=1$ and $b=-2$ are solutions!

b) Set up integrals which give area of the region bounded by the first quadrant and the lines $y=\sqrt{x}$ and $y = x-2$. One integrating with respect to x and the other integrating with respect to y.



First we set off to find the intersection point:

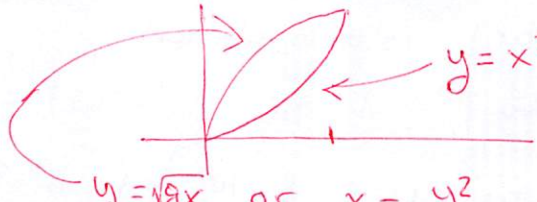
$\sqrt{x} = x-2 \Rightarrow x = (x-2)^2$
$\Leftrightarrow x = x^2-4x+4$
$\Leftrightarrow 0 = x^2-5x+4$
$= (x-4)(x-1)$

We also see that $x=1$ is not a valid solution, and $x=4$ is.

With respect to x: $\int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x} - (x-2)\,dx$

With respect to y: $\int_0^2 y+2-y^2\,dy$

9. (20 points) Let R be the region bounded by $y = \sqrt{8x}$ and $y = x^2$. Using whatever method you like, set up the integral that gives the volume of this region rotated about the given line:



$y = x^2$ or $x = \sqrt{y}$

$x^2 = \sqrt{8x} \iff x = 2$ or $x = 0$

$y = \sqrt{8x}$ or $x = \frac{y^2}{8}$

The x-axis

Washer: $\pi \int_0^2 (\sqrt{8x})^2 - (x^2)^2 \, dx$ Shell: $2\pi \int_0^4 y(\sqrt{y} - y^2/8)\, dy$

The y-axis

Washer: $\pi \int_0^4 (\sqrt{y})^2 - (y^2/8)^2 \, dy$ Shell: $2\pi \int_0^2 x(\sqrt{8x} - x^2)\, dx$

y=-1

Washer: $\pi \int_0^2 (\sqrt{8x}+1)^2 - (x^2+1)^2 \, dx$ Shell: $2\pi \int_0^4 (y+1)(\sqrt{y} - y^2/8)\, dy$

x=-2

Washer: $\pi \int_0^4 (\sqrt{y}+2)^2 - (y^2/8+2)^2 \, dy$ Shell: $2\pi \int_0^2 (x+2)(\sqrt{8x} - x^2)\, dx$

y=5

Washer: $\pi \int_0^2 (5-x^2)^2 - (5-\sqrt{8x})^2 \, dx$ Shell: $2\pi \int_0^4 (5-y)(\sqrt{y} - y^2/8)\, dy$

x=3

Washer: $\pi \int_0^4 (3-y^2/8)^2 - (3-\sqrt{y})^2 \, dy$ Shell: $2\pi \int_0^2 (3-x)(\sqrt{8x} - x^2)\, dx$

Extra Credit: Rotate the region from question 5 about the line $y = -x - 1$ and find the volume of the resulting solid. Note: this is very hard.