# Math 241/251A Final Exam

**Your name:** _____

**Select your instructor and section time:**

- ☐ Luca Candelori (Thursday 1:30pm)
- ☐ Luca Candelori (Friday 10:30am)
- ☐ Erik Guentner (Wednesday 8:30am)
- ☐ Asaf Hadari (Thursday 10:30am)
- ☐ Piper Harron (Thursday 12:00pm)
- ☐ Piper Harron (Friday 9:30am)
- ☐ Mushfeq Khan (Wednesday 10:30)
- ☐ Mushfeq Khan (Wednesday 1:30pm)
- ☐ Daisuke Takagi (Thursday 1:30pm)
- ☐ Daisuke Takagi (Friday 11:30am)
- ☐ David Webb (Friday 8:30)
- ☐ David Yuen (Thursday 8:30am)
- ☐ David Yuen (Thursday 10:30am)

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|---|---|
| 1 (16) | |
| 2 (4) | |
| 3 (10) | |
| 4 (15) | |
| 5 (3) | |
| 6 (10) | |
| 7 (12) | |
| 8 (10) | |
| 9 (10) | |
| 10 (10) | |
| 11 (18) | |
| 12 (6) | |
| 13 (6) | |
| 14 (10) | |
| **TOTAL (140)** | |

**Justify all your work. Answers without suitable justification will receive no credit.**

**Problem 1.** (16 points) Evaluate the following limits. If the limit is infinite, indicate whether it is $\infty$ or $-\infty$. (Do not use l'Hôspital's rule.)

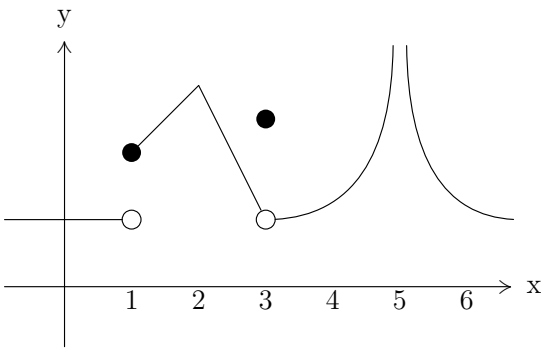
a. $\lim_{x\to\infty} \dfrac{x^3+x}{3x^3-1}$

b. $\lim_{x\to 2^+} \dfrac{4-2x}{|2x-4|}$

c. $\lim_{x\to 0} \dfrac{x^2-4}{x-2}$

d. $\lim_{\theta\to 0} \dfrac{\sin(2\theta)}{\theta}$

**Problem 2.** (4 points) Below is the graph of $y = f(x)$.




a. Find the values of $a$ for which $\lim_{x \to a^+} f(x)$ is infinite or does not exist.

b. Find the values of $a$ for which $\lim_{x \to a^-} f(x)$ is infinite or does not exist.

c. Find the values of $a$ for which $\lim_{x \to a} f(x)$ is infinite or does not exist.

d. Find the values of $a$ for which $f$ is not continuous at $x = a$.

**Problem 3.** (10 points)

a. State the definition of $f'(x)$ as a limit.

b. Let $f(x) = \sqrt{2x}$. Use the definition of the derivative to calculate $f'(2)$ (do not use differentiation rules).

**Problem 4.** (15 points) Find the following derivatives using differentiation rules. You do not have to simplify your answers.

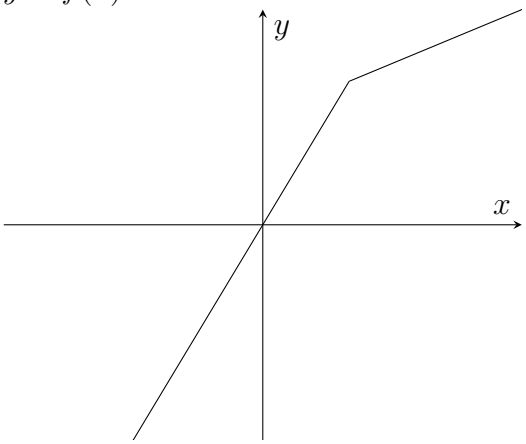
a. $\dfrac{d}{dx}\Big(\sin(x)\tan(x^2)\Big)$

b. $\dfrac{d}{dx}\Big(\dfrac{x}{x^3-1}\Big)$

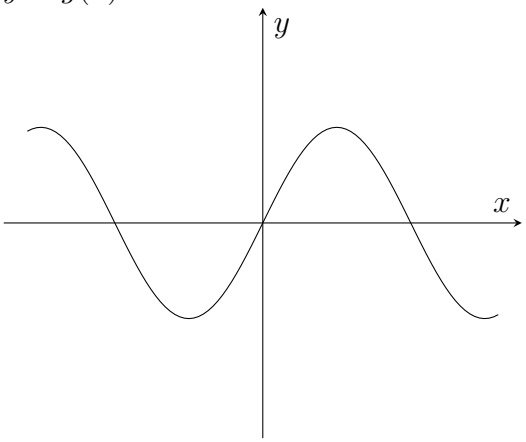
c. $\dfrac{d}{dx}\Big(\sqrt{\cos(2x+1)}\Big)$

**Problem 5.** (3 points) Decide which function on the left has which derivative on the right.

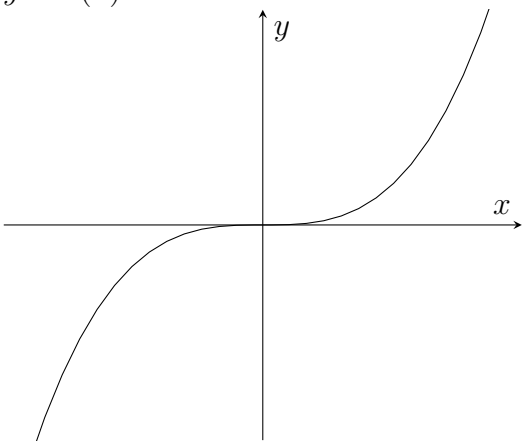
1. $y = f(x)$



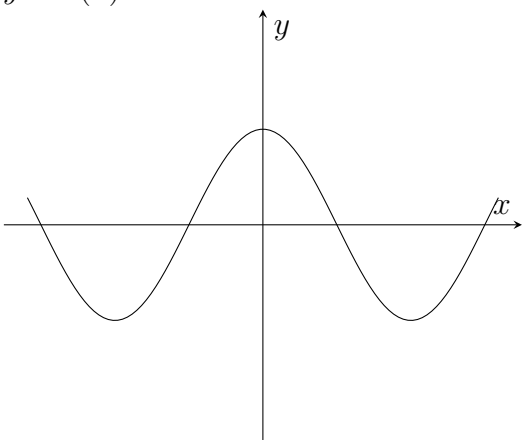

2. $y = g(x)$



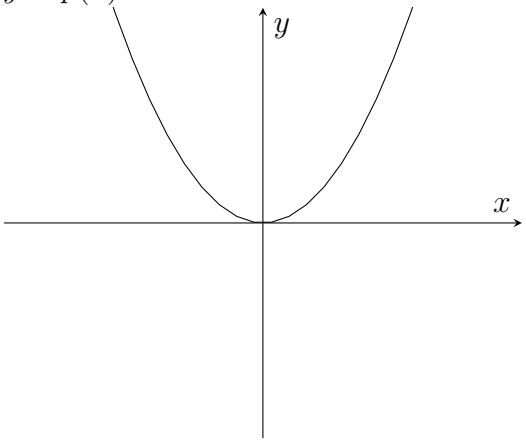

3. $y = h(x)$



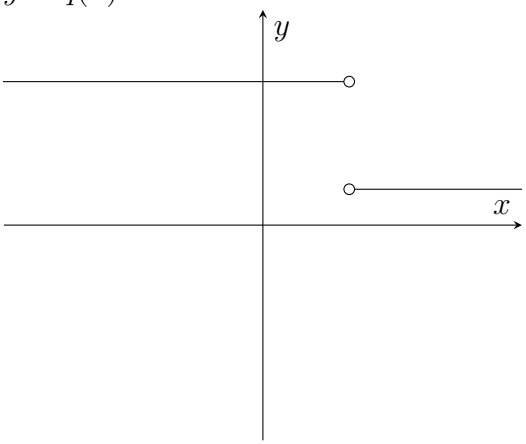

a. $y = k(x)$




b. $y = p(x)$




c. $y = q(x)$




1. $f'(x) =$ ☐ $k(x)$ ☐ $p(x)$ ☐ $q(x)$
2. $g'(x) =$ ☐ $k(x)$ ☐ $p(x)$ ☐ $q(x)$
3. $h'(x) =$ ☐ $k(x)$ ☐ $p(x)$ ☐ $q(x)$

**Problem 6.** (12 points) A cube of ice is melting evenly at a rate of 12 $\mathrm{cm}^3/\mathrm{hour}$. How fast is the side length of the cube changing when the side length is 4 cm?

**Problem 7.** (12 points) Let $f(x) = x^4 - 2x^3$.

a. Find the critical points of $f$ and classify them as local minima, local maxima or neither.

b. On which intervals is $f$ increasing and on which is $f$ decreasing?

c. Find the inflection points of $f$ and the intervals on which it is concave up and those on which it is concave down.

d. Find the absolute maximum and the absolute minimum of $f$ on the interval $[-1, 1]$.

**Problem 8.** (10 points) A rectangular section of a beach reserved for monk seals is being fenced off on three sides (the fourth side borders on the ocean and does not require fencing). If there are $100m$ of fencing, what is the largest area that can be fenced off?

**Problem 9.** (10 points) Find an equation for the tangent line to the curve $x^2y^2 = 9$ at the point $(3, -1)$.

**Problem 10.** (10 points) Show that $f(x) = 2x - \cos(x)$ has exactly one zero in the interval $[-\pi, \pi]$.

a. Show that $f(x)$ has a zero.

b. Use Rolle's Theorem to show that it has exactly one zero.

**Problem 11.** (18 points) Evaluate the following integrals.

a. $\displaystyle\int_0^1 2x\sqrt{x^2+3}\,dx$

b. $\displaystyle\int \sin^2(x)\cos(x)\,dx$

c. Find $f(x)$ such that $f'(x) = \dfrac{2}{x^2}$ and $f(1) = 0$.

**Problem 12.** (6 points) Setup an integral for the area between the curve $y = x^2 + 2x + 1$ and the line $y = x + 1$. You do not need to evaluate the integral.

**Problem 13.** (6 points) Estimate $\int_{-1}^{2}(x^2 + 1)\ dx$ with a Riemann sum using left endpoints of 3 equal subintervals.

**Problem 14.** (10 points) Consider the region between $y = x^2$, the $x$-axis and the line $x = 1$. Find the volume of the solid that is formed by rotating that region around the $y$-axis.