

Problem 1

Determine the critical numbers of $f(x) = x^4 - 4x^3 + 4x^2$. Give intervals of increase/decrease and find any local min/max values by using the first derivative test.

Problem 2

Let $f(x) = 3x^2 - 4x^3$. First find the critical numbers, then find any local min/max values by using the first derivative test.

Problem 3

Determine the critical numbers of $f(x) = x^{2/3}(x^2 - 4)$. Give intervals of increase/decrease, then find any local min/max values by using the first derivative test.

Problem 4

Determine any local min/max values for the function $f(x) = \frac{x^2-3}{x-2}$ by using the first derivative test. Note: $x = 2$ is not a critical number because 2 is not in the domain of $f(x)$, however, $f'(x)$ could change sign at $x = 2$.

Problem 5

Sketch a graph of a function, $f(x)$, with the following properties: $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow \infty} f(x) = 1$,
 $\lim_{x \rightarrow 0^-} f(x) = -\infty$, $\lim_{x \rightarrow 0^+} f(x) = \infty$, $f'(x) > 0$ on $(-\infty, -1) \cup (3, \infty)$, $f'(x) < 0$ on $(-1, 0) \cup (0, 3)$,
 $f''(x) > 0$ on $(-\infty, -2) \cup (0, 4)$, $f''(x) < 0$ on $(-2, 0) \cup (4, \infty)$, $f(3) = -1$ and $f(-1) = 3$.