Worksheet 11 / 15 points

Name: Solutions Section: 5 6

Compute the following:

$$\int_4^9 \frac{1}{2\sqrt{x}} + 2\,dx = \sqrt{x} + 2x \Big|_4^9 = \sqrt{9} + 2\cdot 9 - (\sqrt{4} + 2\cdot 4)$$
$$= 3 + 18 - (2+8)$$
$$= 11$$

$$\int r^2(r^3+1)\,dr$$

$$= \int r^5 + r^2\,dr = \frac{r^6}{6} + \frac{r^3}{3} + C$$

OR

set $u = r^3+1$, then $du = 3r^2dr$
$\frac{1}{3}du = r^2dr$

$$\frac{1}{3}\int u^1 du = \frac{1}{3}\frac{u^2}{2} = \frac{(r^3+1)^2}{6} + C$$

(Note: This is the same as the other answer, but the constant is different)

$$\int \sin(\theta)\cos(\theta)\,d\theta$$

$u = \sin(\theta)$, $du = \cos(\theta)\,d\theta$

$$\int u\,du = \frac{u^2}{2} + C = \frac{\sin^2(x)}{2} + C$$

$$\int \sqrt[3]{\frac{8}{s}}\,ds$$

$$= \int \frac{2}{s^{1/3}}\,ds = 2\int s^{-1/3}ds = 2\,\frac{s^{2/3}}{2/3} + C = 3s^{2/3} + C$$

$$\int_0^1 \frac{1}{\sqrt{3-2x}}\,dx \quad u = 3-2x, \; du = -2\,dx \Rightarrow \frac{-1}{2}du = dx$$

$$\hookrightarrow \frac{-1}{2}\int_3^1 \frac{1}{\sqrt{u}}\,du = \frac{-1}{2}\int_3^1 u^{-1/2}du = \frac{-1}{2}\,\frac{u^{1/2}}{1/2}\Big|_3^1 = -\sqrt{1} + \sqrt{3}$$
$$= -1 + \sqrt{3}$$

$$\int \frac{x^2+x}{(2x^3+3x^2)^3}\,dx \quad u = 2x^3+3x^2, \; du = 6x^2 + 6x\,dx \Rightarrow \frac{1}{6}du = x^2+x\,dx$$

$$\hookrightarrow \frac{1}{6}\int \frac{1}{u^3}\,du = \frac{1}{6}\int u^{-3}du$$
$$= \frac{1}{6}\,\frac{u^{-2}}{-2} + C$$
$$= \frac{-1}{12}\cdot\frac{1}{(2x^3+3x^2)^2} + C$$

Compute the following:

$$\int_0^{\sqrt{\pi/4}} \frac{\sin(x^2)x}{\sqrt{\cos(x^2)}}\,dx \qquad u = \cos(x^2), \quad du = -\sin(x^2)\,2x\,dx$$

$$-\tfrac{1}{2}\,du = \sin(x^2)\,x\,dx$$

$$-\frac{1}{2}\int_1^{\sqrt{2}/2} \frac{1}{\sqrt{u}}\,du = -\frac{1}{2}\int_1^{\sqrt{2}/2} u^{-1/2}\,du$$

$$= -\frac{1}{2}\,\frac{u^{1/2}}{1/2}\Big|_1^{\sqrt{2}/2} = -\sqrt{\frac{\sqrt{2}}{2}} + 1$$

$$\int \sin(\tan(x))\sec^2(x)\,dx$$

$$u = \tan(x), \quad du = \sec^2(x)\,dx$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$= -\cos(\tan(x)) + C$$

$$\int_1^2 \frac{x-2}{\sqrt{x+1}}\,dx \qquad u = x+1, \quad du = dx \quad \text{and ALSO,} \quad \underline{u-3 = x-2}$$

$$= \int_2^3 \frac{u-3}{\sqrt{u}}\,du = \int_2^3 u^{1/2} - 3u^{-1/2}\,du$$

$$= \frac{u^{3/2}}{3/2} - \frac{3u^{1/2}}{1/2}\Big|_2^3 = \frac{3^{3/2}}{3/2} - \frac{3\cdot 3^{1/2}}{1/2} - \left(\frac{2^{3/2}}{3/2} - \frac{3\cdot 2^{1/2}}{1/2}\right)$$

Find the area between $y = \cos(x)$ and $y = 1/2$ on the interval $[0, 2\pi]$.

$$\int_0^{2\pi} \left|\cos(x) - \tfrac{1}{2}\right| dx = \int_0^{\pi/3} \cos(x) - \tfrac{1}{2}\,dx + \int_{\pi/3}^{5\pi/3} \tfrac{1}{2} - \cos(x)\,dx + \int_{5\pi/3}^{2\pi} \cos(x) - \tfrac{1}{2}\,dx$$

These two are =

$$= 2\left(\sin(x) - \tfrac{1}{2}x\right)\Big|_0^{\pi/3} + \tfrac{1}{2}x - \sin(x)\Big|_{\pi/3}^{5\pi/3}$$

$$= 2\left(\sin(\pi/3) - \tfrac{1}{2}(\pi/3)\right) - 2(0)$$

$$+ \tfrac{1}{2}(5\pi/3) - \sin(5\pi/3) - \left(\tfrac{1}{2}\left(\tfrac{\pi}{3}\right) - \sin(\pi/3)\right)$$