Worksheet 3 Lots / 15 points

Name: Batman Section: 5 6 (circle one)

Find the following limits, if they exist:

1. $\lim_{x\to 3^-} \frac{(x-4)|x-3|}{x^2-9} = \lim_{x\to 3^-} \frac{(x-4)(-(x-3))}{(x+3)(x-3)} = \lim_{x\to 3^-} \frac{-(x-4)}{x+3} = \boxed{\frac{1}{6}}$

$x \to 3^-$

$\Rightarrow x < 3$

$\Rightarrow x - 3 < 0$

$\Rightarrow |x-3| = -(x-3)$

2. $\lim_{x\to 0} \frac{\sin(2x)}{3x} = \lim_{x\to 0} \frac{\sin(2x)}{3x} \cdot \frac{2}{2} = \lim_{x\to 0} \frac{1}{3} \frac{\sin(2x)}{2x} \cdot 2$

$= \frac{2}{3} \lim_{x\to 0} \frac{\sin(2x)}{2x} = \frac{2}{3}(1) = \boxed{\frac{2}{3}}$

3. $\lim_{\cancel{x\to 0}} \frac{\sin(x)}{\sqrt{x}} = \lim_{x\to 0^+} \frac{\sin(x)}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \lim_{x\to 0^+} \frac{\sin(x)}{x} \cdot \sqrt{x}$

$\{x \to 0^+\}$

$= 1 \cdot 0 = \boxed{0}$

4. $\lim_{x\to 2} (x-2)^2 \sin\left(\frac{1}{x-2}\right)$ The inequality $-1 \le \sin\left(\frac{1}{x-2}\right) \le 1$ gives

$-(x-2)^2 \le (x-2)^2 \sin\left(\frac{1}{x-2}\right) \le (x-2)^2$.

Since $\lim_{x\to 2} -(x-2)^2 = -(2-2)^2 = 0$

and $\lim_{x\to 2} (x-2)^2 = (2-2)^2 = 0$, we have that $\lim_{x\to 2} (x^2-2)\sin\left(\frac{1}{x-2}\right) = 0$

by the squeeze theorem.

5. $\lim_{x\to\infty} \frac{\sin(2x+1)}{x+3}$

$-1 \le \sin(2x+1) \le 1 \Rightarrow \frac{-1}{x+3} \le \frac{\sin(2x+1)}{x+3} \le \frac{1}{x+3}$ for all $x > -3$.

Since $\lim_{x\to\infty} \frac{-1}{x+3} = \lim_{x\to\infty} \frac{-1/x}{1+3/x} = 0$ and $\lim_{x\to\infty} \frac{1}{x+3} = \lim_{x\to\infty} \frac{1/x}{1+3/x} = 0$,

we have that $\lim_{x\to\infty} \frac{\sin(2x+1)}{x+3} = 0$ by the squeeze theorem.

6. $\lim_{x\to\infty} \frac{\sqrt{x^2+1}+2x}{3x+7} = \lim_{x\to\infty} \frac{\sqrt{x^2(1+1/x^2)} + 2x}{3x+7}$

Note that $|x| = x$ here because $x \to \infty$ and is therefore > 0.

$= \lim_{x\to\infty} \frac{|x|\sqrt{1+1/x^2} + 2x}{3x+7} = \lim_{x\to\infty} \frac{x\left(\sqrt{1+1/x^2}+2\right)}{x(3+7/x)}$

$= \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}+2}{3+7/x} = \frac{\sqrt{1}+2}{3} = 1$