Worksheet 4 A Bunch / 15 points

Name: Batboy Section: 5 6 (circle one)

1. Use the intermediate value theorem to show that the equation $x^3 + 2x + 1 = 0$ has a solution in the interval $[-1, 0]$.

Let $f(x) = x^3+2x+1$. As $f(x)$ is a polynomial, it's continuous. Furthermore, $f(-1) = -1^3 - 2 + 1 = -2$ and $f(0) = 1$, and as $-2 \leq 0 \leq 1$, the IVT gives a c in $(-1, 0)$ such that $f(c) = 0$. Now, $c^3 + 2c + 1 = f(c) = 0$ shows that c is a solution to the equation.

2. Find a value of b so that the function

$$f(x) = \begin{cases} x - b & x < 1 \\ bx^2 + 2 & x \geq 1 \end{cases}$$

is continuous on $(-\infty, \infty)$. We are only worried about continuity at $x = 1$. If $f(x)$ is continuous at $x=1$ then it must be the case that

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x),$$

but this is exactly

$$1 - b = b(1)^2 + 2, \text{ and whence } b = -1/2.$$

3. Using the definition of the derivative as a limit, find $f'(x)$ when $f(x) = \dfrac{1}{x^2+1}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2+1} - \frac{1}{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{x^2+1 - (x^2+2xh+h^2+1)}{h((x+h)^2+1)(x^2+1)}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{h((x+h)^2+1)(x^2+1)}$$

$$= \lim_{h \to 0} \frac{-2x - h}{((x+h)^2+1)(x^2+1)}$$

$$= \frac{-2x}{(x^2+1)^2}$$

Note: now that you are super good at chain rule, you can check your answer... super easy!

4. let $h(x) = 3x^2 + 3\sqrt{x} + \frac{1}{\sqrt[3]{x}}$, find $h'(x)$.

$$h'(x) = 6x + \frac{3}{2\sqrt{x}} + \left(\frac{-1}{3}\right)x^{-4/3}$$

5. Find $\frac{d}{dx}\Big((\sin(x))(5x^{2/3} + 7x^{1/7} + 15)\Big)$. $= \cos(x)\left(5x^{2/3} + 7x^{1/7} + 15\right) + \left(\frac{10}{3}x^{-1/3} + x^{-6/7}\right)\sin(x)$

6. Find $\frac{d}{dt}\left(\frac{8\cos(t) + 2t}{7t^2 + 2t}\right)$ $= \frac{(-8\sin(t) + 2)(7t^2+2t) - (14t+2)(8\cos(t)+2t)}{(7t^2+2t)^2}$

7. Let $g(x) = \frac{3\cos(x) + x^2\sin(x)}{4x^2 + 1}$, find $\frac{dg}{dx}$.

$$\frac{dg}{dx} = \frac{(-3\sin(x) + 2x\sin(x) + x^2\cos(x))(4x^2+1) - (8x)(3\cos(x) + x^2\sin(x))}{(4x^2+1)^2}$$

8. Let $W(x) = \left(\frac{\sin(x)}{x} + \sqrt{x} + 1\right)\left(x^3 + x^2 + x + 1\right)$. Find $W'(x)$.

$$W'(x) = \left(\frac{\cos(x)\,x - \sin(x)}{x^2} + \frac{1}{2\sqrt{x}}\right)\left(x^3+x^2+x+1\right)$$
$$+ \left(3x^2 + 2x + 1\right)\left(\frac{\sin(x)}{x} + \sqrt{x} + 1\right)$$