Worksheet 5 many / 15 points

Name: Batguy Section: 5 6 (circle one)

1. $\frac{d}{dx}\left(\tan(x^2+x)\right) = \sec^2(x^2+x)\,(2x+1)$

2. $\frac{d}{dx}\left(x\sin\left(\frac{1}{x^2}\right)\right) = 1\cdot\sin\left(\frac{1}{x^2}\right) + \cos\left(\frac{1}{x^2}\right)\cdot\frac{-2}{x^3}\cdot x$

3. $\frac{d}{dx}\left(\sec(\sin(x^2))\right) = \sec(\sin(x^2))\tan(\sin(x^2))\cdot\cos(x^2)\cdot 2x$

4. $\frac{d}{dx}\left(\frac{\csc^2(\sqrt{x})}{\sqrt{3x+2}}\right) = \dfrac{2\csc(\sqrt{x})\cdot(-\csc(\sqrt{x})\cot(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}\cdot\sqrt{3x+2} - \frac{1}{2\sqrt{3x+2}}\cdot 3\cdot\csc^2(\sqrt{x})\,)}{(\sqrt{3x+2}\,)^2}$

5. Given a line with slope m, any line perpendicular to it has slope $\frac{-1}{m}$. The **normal** line to a curve (at a point) is defined to be the unique line perpendicular to the tangent line (at the point). If

$$y^3 + xy + x^2 = 3,$$

find the equations of both the tangent and normal lines to the curve at $(1, 1)$.

$$3y^2 \cdot \frac{dy}{dx} + y + x\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx}\left(3y^2 + x\right) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{3y^2+x}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2(1)-1}{3(1)^2+1} = \frac{-3}{4}$$

So, the slope of the tangent line is $\frac{-3}{4}$ and the slope of the normal line is $\frac{4}{3}$.

Tangent line:

$$y - 1 = \frac{-3}{4}(x-1)$$

Normal line:

$$y - 1 = \frac{4}{3}(x-1)$$

6. Suppose an object has a position function $s = p(t) = t^3 - 9t^2 + 24t$ (where s is in meters and t is in seconds. Answer the following questions:

1) When is the object's velocity 0? What is the object's velocity at $t = 0$?

$$V(t) = p'(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$$
$$= 3(t-2)(t-4)$$

so, $V(t) = 0$ when $t = 2$ or $t = 4$

and $V(0) = 24$

2) When is the object moving forward? Backward?

Moving forward when $V(t) > 0$, and this takes place on $(-\infty, 2) \cup (4, \infty)$

Moving backward when $V(t) < 0$, so on $(2, 4)$.

3) What is the object's acceleration at $t = 4$?

$$a(t) = V'(t) = 6t - 18 = 6(t-3).$$

$$a(4) = 6$$

4) When is the object speeding up? $a(t) > 0$ and $V(t) > 0$ OR $a(t) < 0$ and $V(t) < 0$.

$a(t) > 0$ on $(3, \infty)$

and $a(t) < 0$ on $(-\infty, 3)$.

The object is speeding up on $(2, 3) \cup (4, \infty)$