Worksheet 6 ↑↑↑ / 15 points

Name: Catman Section: 5 6 (circle one)

1. Approximate $\sqrt[3]{28}$ with a rational number using linearization.

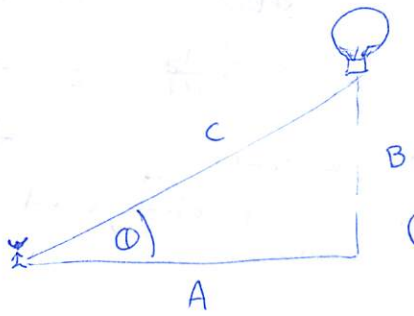
we construct the tangent line to $f(x) = \sqrt[3]{x}$ at $x = 27$:

$f'(x) = \frac{1}{3}x^{-2/3}$, $f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$, and $f(27) = 3$.

so $L(x) = \frac{1}{27}(x-27) + 3$ and therefore

$$\sqrt[3]{28} \approx L(28) = \frac{1}{27}(28-27) + 3$$
$$= 3 + \frac{1}{27}$$

2. A young Batman stands 30 ft. away from a hot air balloon that rests on the ground. He has tied a kite string to the bottom of the hot air balloon. As the balloon rises, Batboy notices that he is letting out string at a rate of 10ft./min at the exact time that he has let out 50 ft. of string. How fast is the balloon rising at this time? How fast is the angle between the ground and the string increasing at this time?



$A = 30$ (fixed/constant)
$B = 40$ (changing)
$C = 50$ (changing)

① $\frac{dC}{dt} = 10$ ft/min, want $\frac{dB}{dt}$:

$A^2 + B^2 = C^2 \xrightarrow{d/dt} 0 + 2B\cdot\frac{dB}{dt} = 2C\frac{dC}{dt}$

$$\frac{dB}{dt} = \frac{C\cdot\frac{dC}{dt}}{B} = \frac{50\cdot 10}{40} = \frac{50}{4} = \frac{25}{2} \text{ ft/min.}$$

② $\tan(\theta) = \frac{B}{A} = \frac{1}{30}B$

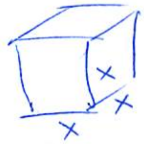
$\Rightarrow \sec^2(\theta)\cdot\frac{d\theta}{dt} = \frac{1}{30}\cdot\frac{dB}{dt}$

and since $\cos(\theta) = \frac{A}{C} = \frac{30}{50} = \frac{3}{5}$, $\sec^2(\theta) = \left(\frac{5}{3}\right)^2$

$$\frac{d\theta}{dt} = \frac{1}{30}\cdot\frac{25}{2}\cdot\left(\frac{3}{5}\right)^2$$
$$= \frac{9}{60} \text{ rad/min.} = \frac{3}{20} \text{ rad/min.}$$

3. Suppose that a cube's volume increases at a constant rate of $2\frac{cm.^3}{min.}$. How fast are it's sides growing at the time the sides are $10cm.$? How fast is the surface area increasing at this time?

$$V = x^3$$



Given: $\frac{dV}{dt} = 2\ cm^3/min$

$SA = 6x^2$, so,

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

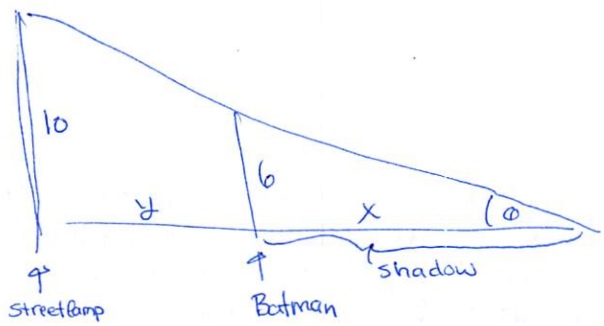
$$\Rightarrow \frac{dx}{dt} = \cancel{\frac{3x^2}{3x^2}} \frac{2}{3(10)^2} = \frac{2}{300}$$

$$\frac{dSA}{dt} = 12x \cdot \frac{dx}{dt}$$

$$= 12 \cdot 10 \cdot \frac{2}{3 \cdot 10^2}$$

$$= 4 \cdot 2/10 = \frac{4}{5}\ cm^2/min$$

4. A 6 ft tall Batman runs (into the night) away from a 10 ft streetlamp at a rate of $1\frac{\cancel{m}\,ft}{sec}$. How fast is his shadow growing when he is 5 ft away from the lamp? How fast is the angle between the ground and the top streetlamp changing at the tip of his shadow?



(1) $\frac{6}{x} = \frac{10}{x+y}$

$\Rightarrow 6x + 6y = 10x$

$6y = 4x$

$x = \frac{3}{2}y \quad (*)$

Given: $\frac{dy}{dt} = 1\ ft/sec.$

$\frac{dx}{dt} = \frac{3}{2}\frac{dy}{dt} = \frac{3}{2}\ ft/sec.$

Notice that $\frac{dx}{dt}$ is the rate of change of his shadow

(2) ∡ $\tan(\theta) = \frac{6}{x} = 6x^{-1}$

Also, from $(*)$ we know that $x = \frac{15}{2}$.

Now,

$$\frac{d\theta}{dt} \sec^2(\theta) = -6x^{-2} \cdot \frac{dx}{dt}$$

$$\text{so, } \frac{d\theta}{dt} = \frac{-6}{\left(\frac{15}{2}\right)^2} \cdot \frac{3}{2} \cdot \frac{1}{\left(\frac{\sqrt{\left(\frac{15}{2}\right)^2+6^2}}{15/2}\right)^2}$$

$$= \frac{-18}{2\left(\sqrt{\left(\frac{15}{2}\right)^2+6^2}\right)^2} = \frac{-9}{\left(\frac{15}{2}\right)^2+6^2}\ rad/sec$$

(I'm stopping here)