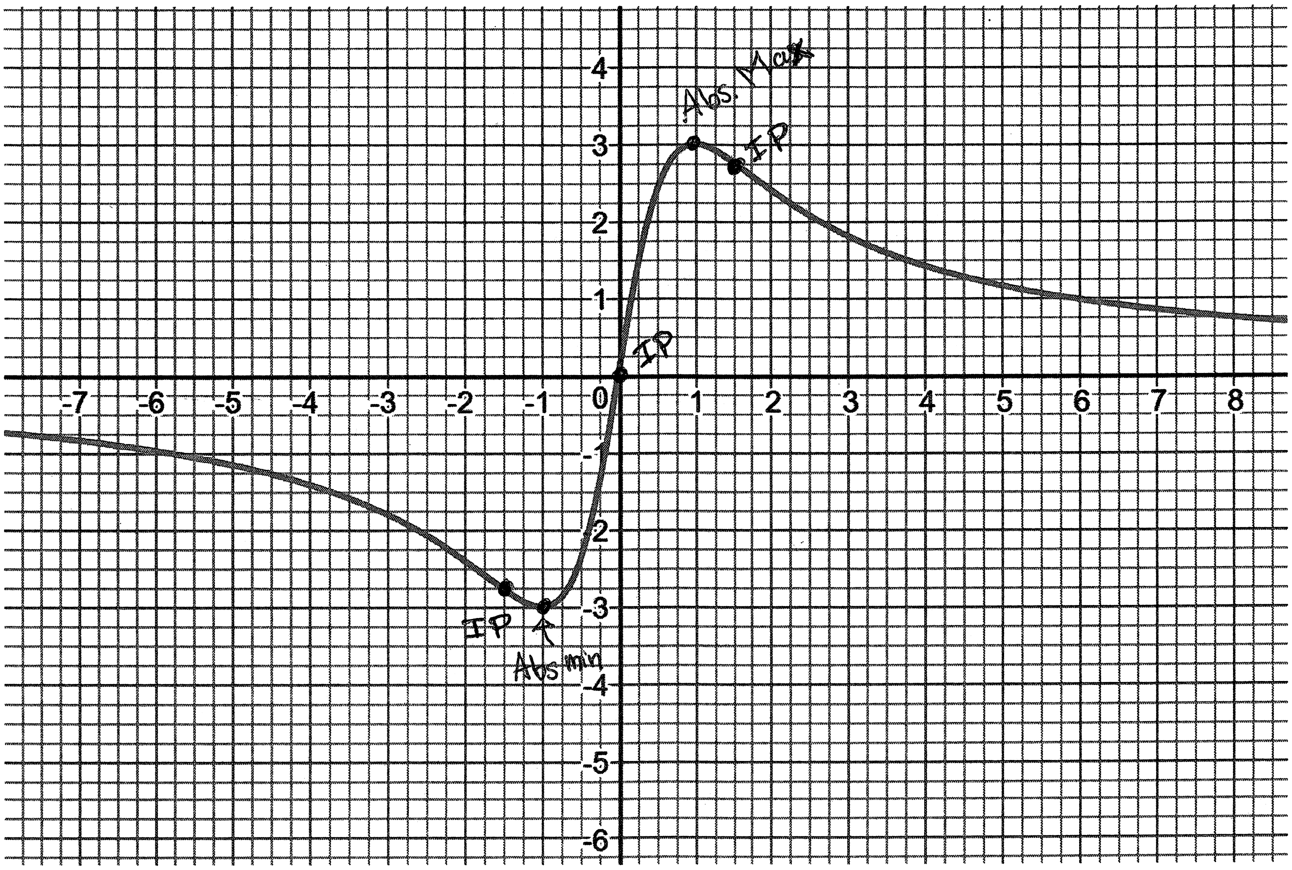
Worksheet 8 200 / 15 points

Name: Graphman Section: 5 6

Here is a graph of the function $y = \dfrac{6x}{x^2+1}$:



Find the exact values (using calculus) of the intervals of increase, decrease and concavity, absolute min/max, limits at $\pm\infty$, inflection points and label the graph.

$$y' = \frac{6(x^2+1) - 2x(6x)}{(x^2+1)^2}$$

$$= \frac{6(1-x^2)}{(x^2+1)^2}$$

y': $-$ on $(-\infty,-1)$, $+$ on $(-1,1)$, $-$ on $(1,\infty)$; marks at -1, 1

$$y'' = \frac{-12x(x^2+1)^2 - 2(x^2+1)\cdot 2x\,(6(1-x^2))}{(x^2+1)^4}$$

$$= \frac{-12x(x^2+1)\left[x^2+1+2(1-x^2)\right]}{(x^2+1)^4}$$

$$= \frac{-12x(3-x^2)}{(x^2+1)^3}$$

y'': $-$, $+$, $-$, $+$; marks at $-\sqrt{3}$, 0, $\sqrt{3}$

$$\lim_{x\to\infty} \frac{6x}{x^2+1} = 0$$

$$\lim_{x\to-\infty} \frac{6x}{x^2+1} = 0$$

Increase: $(-1,1)$

Decrease: $(-\infty,-1) \cup (1,\infty)$

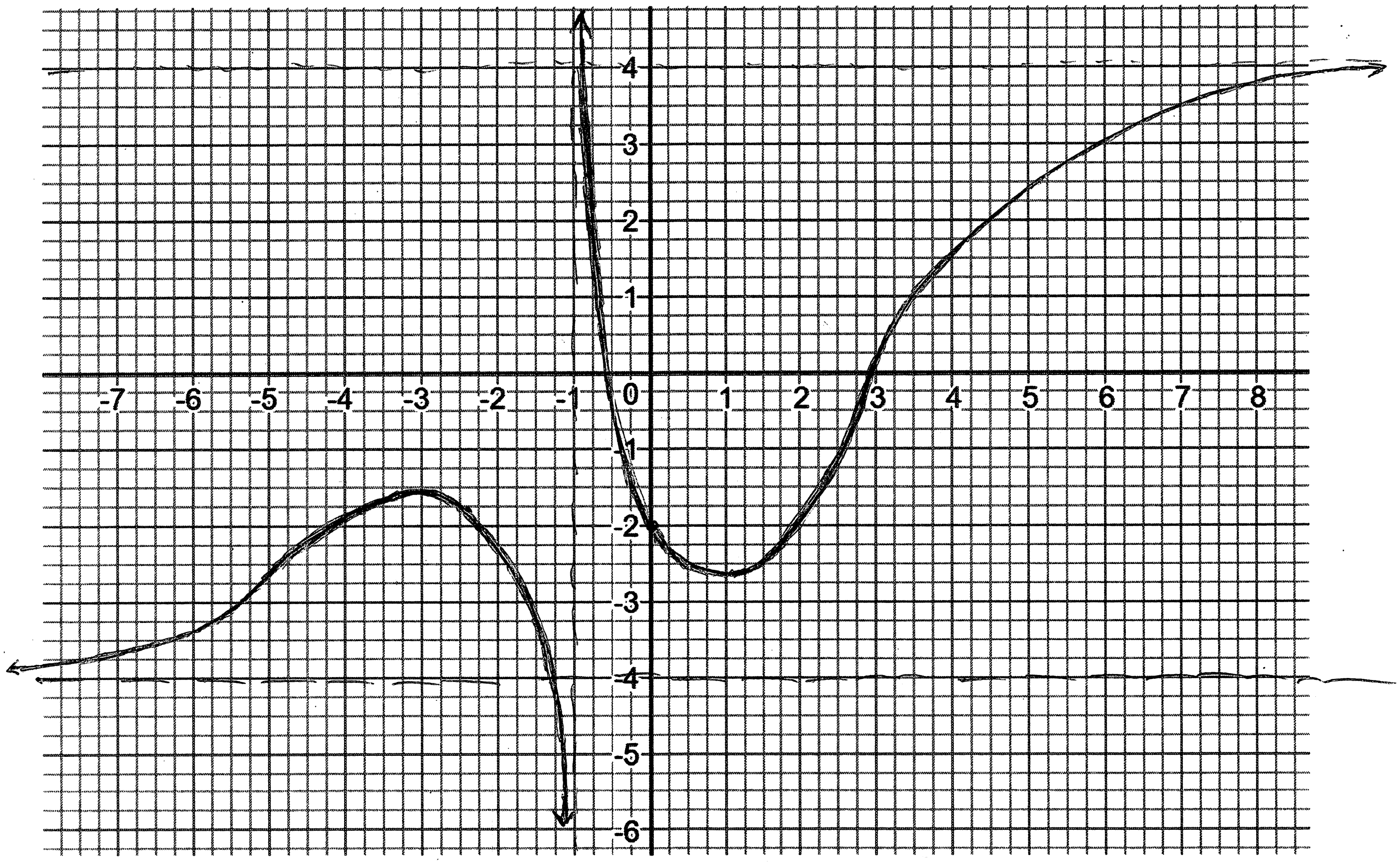
Concave up: $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$

Concave down: $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$

Inflection points @ $\left(-\sqrt{3}, \frac{-6\sqrt{3}}{4}\right)$, $(0,0)$ and $\left(\sqrt{3}, \frac{6\sqrt{3}}{4}\right)$

local min @ $(-1,-3)$,

local max @ $(1,3)$.



In the above space, draw a graph of a function with the following properties:

$f(0) = -2$, $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = -4$, $f(x)$ has a vertical asymptote at $x = -1$, $f'(x) > 0$ for $-\infty < x < -3$ and $1 < x < \infty$, $f'(x) < 0$ for $-3 < x < -1$ and $-1 < x < 1$, $f''(x) > 0$ for $-\infty < x < -5$ and $-1 < x < 3$, $f''(x) < 0$ for $-5 < x < -1$ and $3 < x < \infty$.

Draw a goat.