Math 241
Spring 2018
Final
5/9/2018
Time Limit: 120 minutes

Name: ______________________________

Please read carefully

- No calculators or notes are allowed.
- Show your work.
- When applicable, indicate your final answer by drawing a box around it.
- One side of each sheet is blank and may be used for scratch work. Scratch work will not be graded.
- Circle your instructor and section number:

Robertson 1	Lyons 4	Hadari 7
Antin 2	Lyons 5	Harron 8
Antin 3	Hadari 6	Harron 9

Grade Table (for instructor use only)

Question	Points	Score
1	48	
2	32	
3	8	
4	8	
5	8	
6	8	
7	8	
8	14	
9	32	
10	10	
11	12	
12	12	
Total:	200	

1. Differentiate the following functions. You do not have to simplify your answer.

   (a) (8 points) $y = \dfrac{2}{x^3} + \dfrac{x^3}{2}$

   (b) (8 points) $y = \dfrac{1}{2 + x^2}$

   (c) (8 points) $y = \cos\left(\dfrac{x}{2}\right) \tan x$

(d) (8 points) $y = \sin(3\sqrt{1-x})$

(e) (8 points) $y = \dfrac{\sqrt[3]{x} - 1}{x - 1}$

(f) (8 points) $y = \displaystyle\int_x^0 \sqrt{1+t^2}\, dt$

2. For each of the following limits, if it exists, compute it. If the limit is infinite, say whether it is ∞ or $-\infty$. If it does not exist, explain why not.

   (a) (8 points) $\lim_{x \to 2} \frac{x^3 - 6x + 4}{x + 2}$

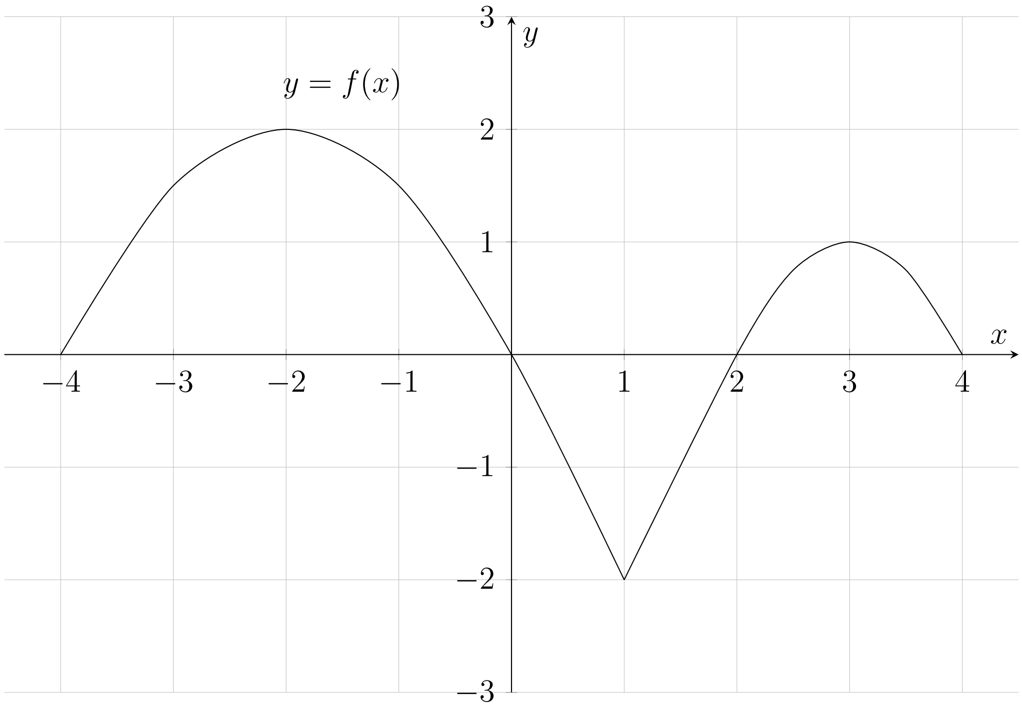
   (b) (8 points) $\lim_{x \to 3} \frac{x - 3}{\sqrt{x} - \sqrt{3}}$

   (c) (8 points) $\lim_{x \to \infty} \frac{x^3 + \cos x}{3 - x^3}$

   (d) (8 points) $\lim_{x \to 0} x^{-\frac{2}{3}}$

3. (8 points) Using the **limit definition of the derivative**, find $f'(x)$ if $f(x) = \dfrac{x+1}{x}$.

4. (8 points) The following is the graph of a function f on the domain $[-4, 4]$.

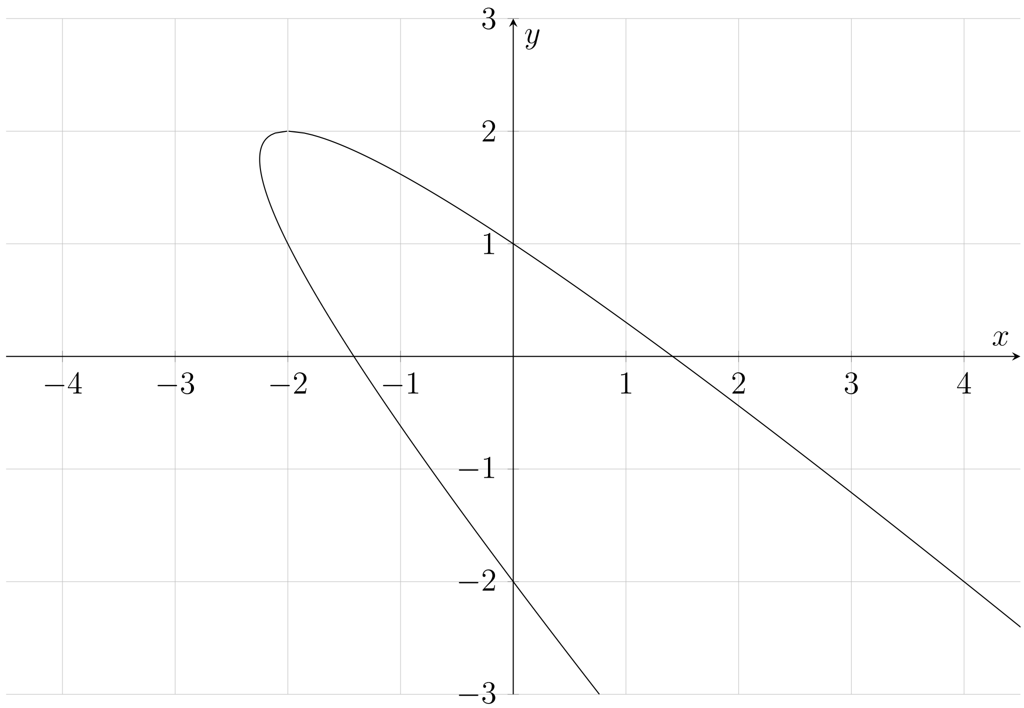


On the same set of axes, sketch the graph of the derivative f'. If there are discontinuities, indicate them clearly.

5. The equation

$$(y+x)^2 = 2 - y$$

describes the following curve in the plane:



(a) (6 points) Find the equation of the tangent line to this curve at the point $(0, 1)$.

(b) (2 points) On the graph above, sketch the tangent lines to the curve at the points $(0, 1)$ and $(-2, 2)$.

6. You're pumping air into a spherical balloon at a steady rate of $5\,\text{cm}^3/\text{s}$.

   (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ while its surface area is $4\pi r^2$.)

   (a) (4 points) At what rate is its radius increasing (in cm/s) when its radius is 3 cm?

   (b) (4 points) At what rate is its surface area increasing (in cm^2/s) when its radius is 3 cm?

7. (8 points) A designer wants to make a new line of bookcases. They want to make at least 100 of them and not more than 1000. They predict that the cost of producing x bookcases is $C(x) = 42(x^2 + 400)$ dollars. Find the number of bookcases that will minimize the **average cost** $A(x)$, given by $C(x)/x$.

8. Let

$$f(x) = \frac{1}{1+x^2}.$$

Then

$$f'(x) = -\frac{2x}{(1+x^2)^2} \qquad \text{and} \qquad f''(x) = \frac{6x^2-2}{(1+x^2)^3}.$$

(a) (4 points) Find the intervals on which the graph of f is increasing and those on which it is decreasing. Find the extrema, if there are any, and determine which of them are absolute.

(b) (4 points) Find the intervals on which the graph of f is concave up and those on which it is concave down. Find the points of inflection, if any exist.

(c) (2 points) Find the asymptotes, if there are any.

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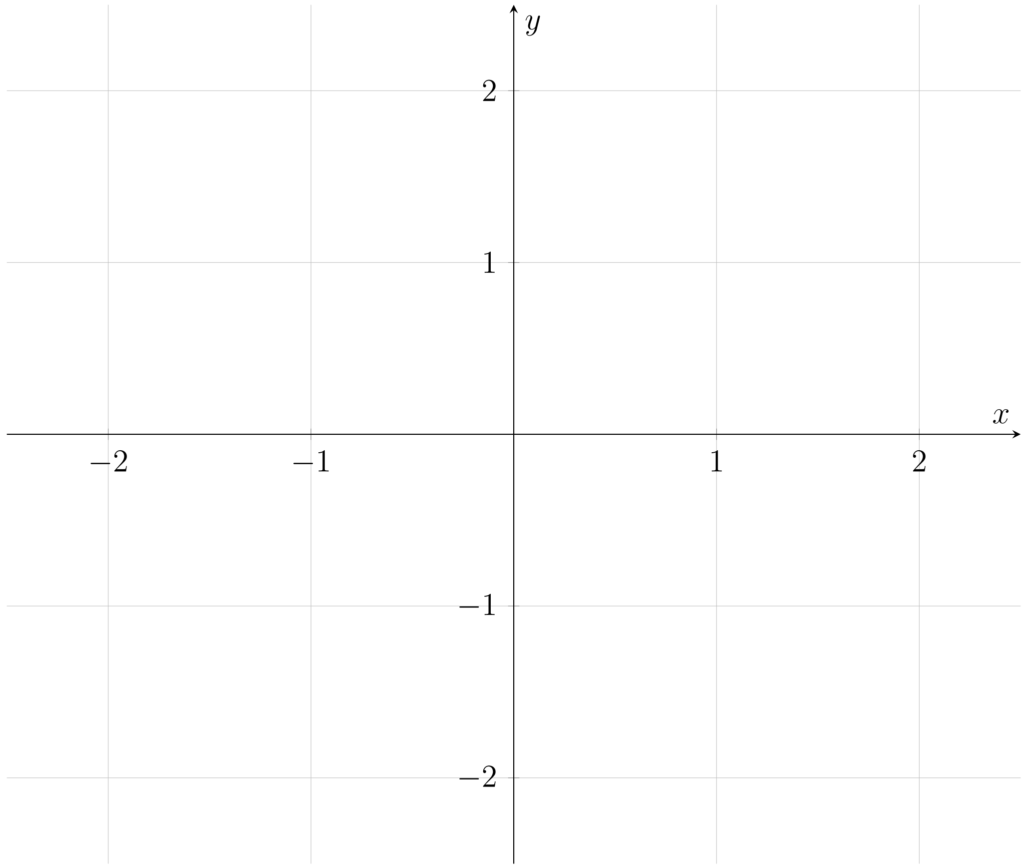
(d) (4 points) Sketch the graph on the axes below. Label the asymptotes, extrema, and points of inflection, if there are any.



9. Compute the following integrals. Simplify your answers.

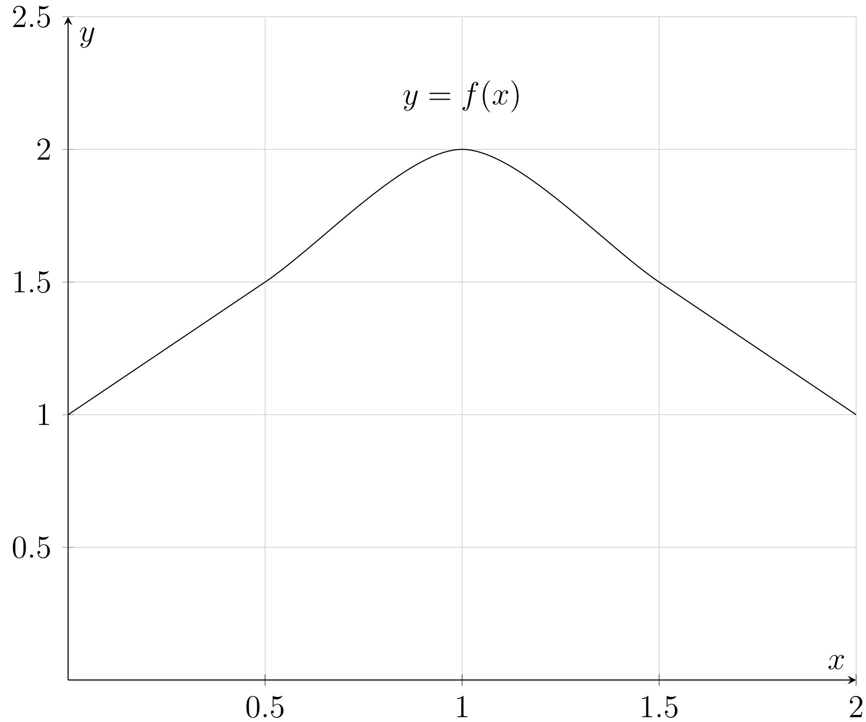
   (a) (8 points) $\displaystyle\int_0^{\pi/2} \cos(2\theta)\, d\theta$

   (b) (8 points) $\displaystyle\int \sqrt[3]{2} - x^{2/3}\, dx$

   (c) (8 points) $\displaystyle\int \frac{\sin 2x}{\cos^7 2x}\, dx$

   (d) (8 points) $\displaystyle\int_0^{\sqrt{8}} \frac{x}{\sqrt{1+x^2}}\, dx$

10. The following is the graph of a function f on the interval $[0, 2]$:

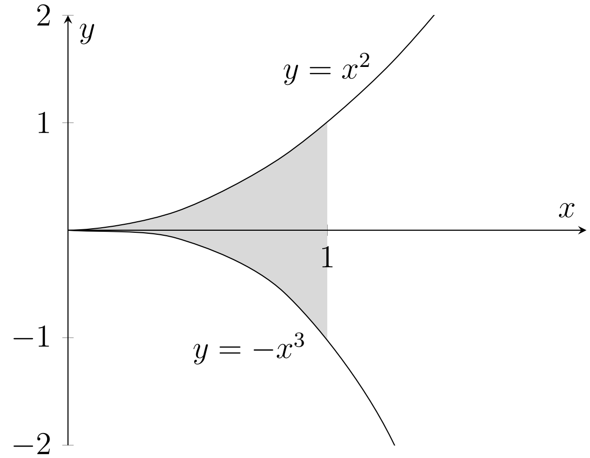


You are asked to compute an approximation to $\int_0^2 f(x)\,dx$ by means of a **lower Riemann sum using 4 subintervals of equal width**.

(a) (5 points) On the graph above, draw the rectangles you should use.

(b) (5 points) Compute the Riemann sum.

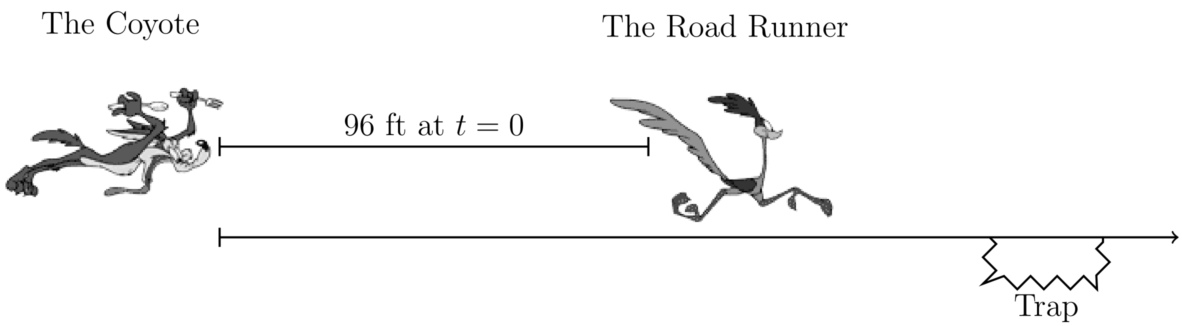
11. Consider the region enclosed by the graphs of $y = x^2$ and $y = -x^3$ between $x = 0$ and $x = 1$:



(a) (6 points) Compute the area of this region.

(b) (6 points) Using the shell method, compute the volume of the solid obtained by revolving this region around the y-axis.

12. The Road Runner is running in a straight line and the Coyote is chasing after him. Suppose that at time $t = 0$, the Road Runner has a head start of 96 ft. The Road Runner's speed is a constant 4 ft/sec, while the Coyote's is $4t$ ft/sec.



(a) (3 points) Find the Road Runner's position as a function of time.

(b) (3 points) Find the Coyote's position as a function of time.

(c) (3 points) How long would it take for the Coyote to catch up with the Road Runner?

(d) (3 points) The Road Runner has set a trap for the Coyote 120 ft from the Coyote's starting position. Does the Coyote reach the trap before he is able to catch the Road Runner?