Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	35	
7	60	
Total:	145	

1. (10 points) For differentiable vector valued functions $u(t)$ and $v(t)$, prove that $\frac{d}{dt}\left(u(t)\cdot v(t)\right) = \frac{du}{dt}\cdot v(t) + u(t)\cdot \frac{dv}{dt}$

2. (10 points) Show that if $r(t)$ is a differentiable vector valued function and $|r(t)| = C$ for a constant C, then $r(t)$ and $\frac{dr}{dt}$ are orthogonal.

3. (10 points) Find $r(t)$ if

$$\frac{d^2r}{dt^2} = -32k, \quad r(0) = 100k, \quad \left.\frac{dr}{dt}\right|_{t=0} = 8\,i + 8\,j$$

4. (10 points) Let $r(t) = t\sin(t^2)\ i + \dfrac{1}{1+t^2}\ j + t\sin(t)\ k$. Find $\int r(t)dt$.

5. (10 points) With $r(t)$ from the previous problem, find $\int_0^{\sqrt{\pi}} r(t)dt$.

6. Let $r(t) = t\cos(t)\ i + t\sin(t)\ j + \dfrac{2\sqrt{2}}{3}t^{3/2}\ k$.

(a) (10 points) In a few words or a sketch, describe this curve for $t \geq 0$.

(b) (10 points) Find the parametric equations of the tangent line to the curve when $t = \frac{\pi}{3}$.

(c) (15 points) Find the length of the curve from $t = 0$ to $t = \pi$.

7. (60 points) For numbers $a, b \geq 0$, let

$$r(t) = a\cos(t)\ i + a\sin(t)\ j + bt\ k.$$

Find the unit tangent vector, T, the principle unit normal vector, N, the curvature, κ, the unit binormal, B, and the torsion, τ, of this curve. Give the equation of the osculating plane at $t = 0$.