Math 243
Spring 2019
Practice Exam 2
Doomsday
Time Limit: Probably Not Enough

Name (Print): ____________________

Problem	Points	Score
1	0	
2	15	
3	20	
4	20	
5	20	
6	15	
7	15	
8	20	
9	20	
10	20	
Total:	165	

1. For the following functions, give the domain and range. Determine if the domain is open or closed (or neither), and determine if the domain is bounded or unbounded.

   a) $f(x, y) = \sqrt{x - y}$

   b) $g(x, y) = x^2 + y^2 - 3$

   c) $h(x, y) = \sqrt{25 - x^2 - y^2}$

2. (15 points) Let $f(x,y) = \frac{1}{16-x^2-y^2}$

a) Find the domain and range of $f(x,y)$. Note: The range is tricky.

b) Is the domain open/closed or neither? What is the boundary of the domain? Is the domain bounded or unbounded?

c) Graph the level curves $f(x,y) = \frac{1}{15}$ and $f(x,y) = 1$. Include the vector $\nabla f|_{(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})}$ on the appropriate level curve.

3. Consider the function $f(x, y, z) = y^2 - x^2 - z$.

   (a) (10 points) Graph the level surface $f(x, y, z) = 0$.

   (b) (10 points) Suppose we stood on this surface at the point $(1, 1, 0)$. We the trail we are on goes directly up the surface in the "steepest" direction. What is the direction and how steep is the trail?

4. (a) (10 points) Find $\lim_{(x,y)\to(2,2)} \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$ if it exists.

(b) (10 points) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - y^2}{x^4 + y^2}$ if it exists.

5. (a) (10 points) Find $\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\left(\dfrac{y^2+\sin(y)}{y^4}+x\right)\right)$

(b) (10 points) Let $f(x,y,z)=\dfrac{ye^{xyz}}{x}$. Find f_x, f_y and f_z. Then, find f_{xyz}.

6. (15 points) a) Let $f(x, y)$ be a function with continuous partial derivatives. Suppose that $r(t) = g(t)\ i + h(t)\ j$ is a (differentiable) vector valued function and $f(g(t), h(t)) = c$ for some constant c. Show that ∇f and $\frac{dr}{dt}$ are orthogonal along this level curve.

b) Find the the derivative of $f(x, y) = \ln(x^2 + y^2)$ in the direction of $v = i + j$ at the point $(1, 1)$.

7. (15 points) a) Let $z = x^2 - y^2 + 3$. Find the equation of the tangent plane at the point $(1, 1, 3)$.

b) Let $f(x, y, z) = x^3 + y^2 + 3z + 4$. On the level surface $f(x, y, z) = 0$, give the equation of the tangent plane at the point $(1, 2, -3)$.

c) The surface $x^2 + y^2 = 4$ is "sliced" by the plane $x + y + z + 1 = 0$ and forms an ellipse. Find the parametric equations for the tangent line to this ellipse at the point $(2, 2, -5)$.

8. (a) (20 points) Let $f(x, y) = 9x^3 + y^3/3 - 4xy$. Use the second derivative test to find any local min/max or saddle points.

9. (20 points) Find the absolute maxima and minima of $f(x, y) = x^2 - xy + y^2 + 1$ on the closed region bounded by $x = 0$, $y = 4$ and $y = x$.

10. (20 points) Find the point on the plane

$$2x + 2y + 2z = 2$$

which is closest to the origin.