Problem 1

Find the derivative of $f(x, y) = x^2 + 3xy + y^2$ in the direction of $v = 2i + j$.

Problem 2

Define $f(x, y) = x^2 - y^2$. Sketch the level curve containing the point $(0, 1)$. Compute ∇f. Plot the tangent line and ∇f on the level curve at this point.

Problem 3

Let $f(x, y) = x^2y + e^{xy}\sin(y)$. At the point $(1, 0)$, in what direction does f increase most rapidly? Find the derivative of f in this direction.

Problem 4

Let $f(x, y) = \ln(x^2 + y^2 - 1) + y + 6z$. At the point $(1, 1, 0)$, find the direction in which f increases mostly rapidly. Then find the derivative of f in that direction.

Problem 5

Let $f(x, y) = xy + y^2$. Find a direction, u, such that $(D_u f)|_{(3,2)} = 0$. In what direction does f **decrease** most rapidly? Find the derivative of f in this direction.

Problem 6

Let $f(x, y, z) = 2x^2 + y^2 - 3z^2$. Show that $(1, 1, 1)$ is on the level surface $f(x, y, z) = 0$. Find an equation of the tangent plane to the level surface $f(x, y, z) = 0$ at the point $(1, 1, 1)$. Give parametric equations of the normal line to the surface at this point.

Problem 7

Let $f(x, y) = e^{xy}x^2 + y^2$. Determine an equation of the tangent plane on the surface $z = f(x, y)$ when $(x, y) = (1, 1)$. Give an equation of the normal line to the surface at this point.

Problem 8

Find an equation for the plane that is tangent to the surface $z = e^{x^2+y^2} + 2xy - x^2 - y^2$ at the point $(1, 1, e^2)$.

Problem 9

Consider the surfaces $x^3 - xyz + y^3 = 1$ and $x^2 + y^2 + z^2 = 3$. Find parametric equations for the line which is tangent to the curve of intersection at the point $(1, 1, 1)$.

Problem 10

Let f and g be functions of 2 variables. Show that

$$\nabla(fg) = g\nabla f + f\nabla g \quad \text{and} \quad \nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}.$$