Math 243
Spring 2019
Practice Exam 1
Doomsday
Time Limit: Probably Not Enough

Name (Print): ____________________

Problem	Points	Score
1	15	
2	30	
3	40	
4	15	
5	20	
6	20	
7	20	
8	10	
9	10	
10	25	
11	40	
Total:	245	

1. (15 points) Let $P = (1, 2, 3)$ and $Q = (0, -1, 2)$.

   a) Find the distance between P and Q.

   b) Give the equation of a sphere, centered at P, that has the point Q on its surface.

   c) Find the vector $\overrightarrow{PQ}$.

   d) Find the angle between $\overrightarrow{P}$ and $\overrightarrow{Q}$.

   e) Parametrize (with parametric equations) the line **segment** which starts at P and ends at Q.

2. The following points define the vertices of a triangle:

$$A = (0, 0, 1) \quad B = (1, 3, -1) \quad C = (2, 2, 2)$$

(a) (10 points) Find the area of the triangle.

(b) (10 points) Find an equation of the plane which contains the triangle.

(c) (10 points) Find the distance between the point $(3, 2, 1)$ and the plane from part b).

3. (a) (10 points) Let $u = i + j$, $v = i + j + k$. Find the projection of u onto $u + v$.

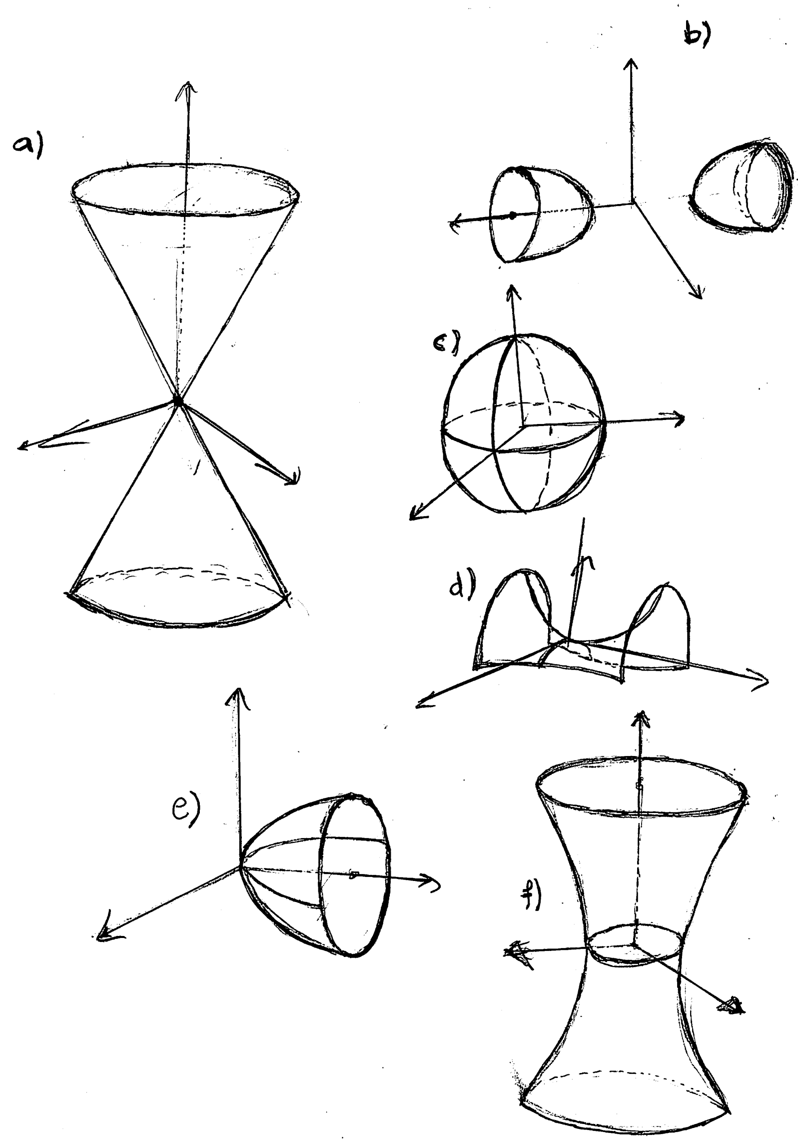
(b) (10 points) The vectors $u = i + 2j$ and $v = j + 3k$ lie in a plane that goes through the point $P = (1, 0, 1)$. Give the equation of this plane.

(c) (10 points) Give the equation of a line, perpendicular to the plane $2x + 3y + z = 6$, that goes through the point $(1, 0, 1)$.

(d) (10 points) Find the point in space the line from part c) intersects the plane from part b).

4. (15 points) Match the surface with its equation

$$
\begin{aligned}
1 &= x^2 + y^2 - z^2 \\
y &= x^2 + z^2 \\
1 &= x^2 + y^2 + z^2 \\
z &= y^2 - x^2 \\
0 &= x^2 + y^2 - z^2 \\
x^2 &= z^2 + y^2 + 1
\end{aligned}
$$



5. (20 points) Find the length of the curve given by the parametric equations

$$x = \cos(t) \quad \text{and} \quad y = t + \sin(t) \quad \text{for } 0 \leq t \leq \pi.$$

Also, find the equation of the tangent line when $t = \frac{\pi}{2}$.

6. (20 points) Find the length of the curve

$$x = \frac{y^3}{6} + \frac{1}{2y}$$

from $y = 2$ to $y = 3$.

7. (20 points) Consider the polar coordinate equations $r = 2(1 + \cos(\theta))$ and $r = 2(1 - \cos(\theta))$.

   a) Graph both of these curves. For both graphs, find and plot the equation of the tangent line when $\theta = \frac{\pi}{6}$.

b) Find the length of each curve.

c) Find the area between the curves in the first quadrant.

8. (10 points) Show that if $r(t)$ is a differentiable vector valued function and $|r(t)| = C$ for a constant C, then $r(t)$ and $\frac{dr}{dt}$ are orthogonal.

9. (10 points) Find $r(t)$ if

$$\frac{d^2r}{dt^2} = -32k, \quad r(0) = 100k, \quad \left.\frac{dr}{dt}\right|_{t=0} = 8\ i + 8\ j$$

10. Let $r(t) = t\cos(t)\ i + t\sin(t)\ j + \frac{2\sqrt{2}}{3}t^{3/2}\ k$.

(a) (10 points) Find the parametric equations of the tangent line to the curve when $t = \frac{\pi}{3}$.

(b) (15 points) Find the length of the curve from $t = 0$ to $t = \pi$.

11. (40 points) For numbers $a, b \geq 0$, let

$$r(t) = a\cos(t)\ i + a\sin(t)\ j + bt\ k.$$

Find the unit tangent vector, T, the principle unit normal vector, N, the curvature κ, the unit binormal B, and the torsion τ of this curve. Give the equation of the osculating plane at $t = 0$.