Problem 1

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 2y \\ x - y \end{bmatrix}$$

a) Let α be the standard basis for $\mathbb{R}^2$ and let β be the basis $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Find $[T]_{\alpha}^{\beta}$.

b) Find $[T]_{\beta}^{\beta}$.

c) Find the change of basis matrix from α to β, $[I]_{\beta}^{\alpha}$.

d) Find the change of basis matrix from β to α, $[I]_{\alpha}^{\beta}$.

e) Verify that $[T]_{\beta}^{\beta} = P^{-1}[T]_{\alpha}^{\alpha}P$ where $P = [I]_{\beta}^{\alpha}$.

Problem 2

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2y - z \\ 2x - y + z \\ x + z \end{bmatrix}$$

Let α be the standard basis for $\mathbb{R}^3$, and β be the basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$. Find $[T]_\beta^\beta$.

Problem 3

Let α be the standard basis for P^3, and $D : P^3 \to P^3$ be the differential operator. Find $[D]_\alpha^\alpha$.